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Fit for Leverage - Modelling of Hedge Fund Returns in View of Risk Management Purposes

by

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Fit for Leverage

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Modelling of Hedge Fund Returns in View of Risk Management Purposes

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Abstract

Hedge funds typically reveal some statistical properties like serial correlation, non-normality, volatility clustering, and leverage effect, which have to be considered when risk positions of hedge funds are computed. We describe an autoregressive Markov-Switching model that captures the specific features of hedge fund returns and allows especially to fit for volatility clustering and leverage effects in the data. The model is tested using publicly available hedge fund index data from different regions. We compare two different variants of the model by means of risk and performance measures. Our case study implies that if the leverage effect appears in the data, it is worth to fit for leverage in the parameter estimation process.

Keywords: Hedge Fund, Leverage Effect, Markov-Switching Autoregressive Model

1 Introduction

The hedge fund industry has been clearly growing during the last twenty years. According to [28], the number of hedge funds (HF) shot up from 610 in 1990 to more than 9,400 in 2006. The estimated assets held by the funds have grown from 39 billion US$ in 1990 over 500 billion US$ in 2000 to about 1.5 trillion US$ in 2006. Whereas 20 years ago most hedge funds were US based, the hedge fund industries in other regions have been strongly increasing in the last years. E.g., the assets under management of Asia-focused hedge funds have grown strongly over the years from barely 5 billion US$ in 1996 to an estimated 170 billion US$ in 2008, with growth of 30 % per annum over the past five years. At the beginning of 2008 some analysts forecasted that Asia, which had around 1,200 hedge funds at this time, will have close to 1 trillion US$ in assets in five years' time (see [33]). Due to the impact of the current financial market crisis this size will probably not be reached. Recent estimates even predict a shrinkage of the whole hedge fund industry between 10% and 50%. Nevertheless, since their invention hedge funds always had an important impact on the financial markets and probably will also have in the future.

Hedge funds often play the role of a trigger and multiplier of crises and problems. For example, Asian hedge funds played a notable role in the Asian Currency Crisis of 1997 (see, e.g., [14]). Another failure of appropriate risk management is the downturn of the US-based Long Term Capital Management (LTCM) hedge fund in 1998 that was due to an underestimation of risk (see, e.g., [22]). Also in the recent subprime crisis hedge funds played a crucial role. This illustrates the importance of a sound risk management and strongly emphasizes a risk-adjusted performance measurement.

There have been different methods proposed for the modelling of hedge fund returns, which can be basically categorized in two categories: replication methods and econometric models.

Factor models, where the return of a particular hedge fund or hedge fund index is attributed to a number of risk factors, are the most common replication models used in academic literature, see e.g. [30], [15], and [20]. The latter constructed a six-factor model with factors describing the stock market, the bond market, currencies, commodities, credit, and volatility and used it to replicate the returns of 1,610 individual hedge funds.

In addition to academic literature, a number of investment banks have recently launched hedge fund replication products based on factor models: In September 2006 Merrill Lynch launched its Factor Index (Bloomberg ticker: MLEIFCTR). In December 2006 Goldman Sachs announced its Absolute Return Tracker (ART) Index (Bloomberg ticker: ARTIUSD), while in February 2007 JP Morgan announced the upcoming launch of its Alternative Beta Index (ABI).

A drawback of a simple factor model is that it has serious difficulty producing accurate replicas for individual hedge funds and most hedge fund indices. To obtain an accurate replication, the factor model approach needs to be applied to an extremely well diversified index, where essentially everything that makes hedge funds interesting, and thereby causes factor models to fail, has been diversified away (see [23] and [16]).

Another way to obtain a sound model for hedge fund return time series is the application of econometric models accounting for typical properties of hedge fund return time series, such as data biases, non-normality, and autocorrelation, which are the most commonly known and observed ones.

Hedge fund returns, like private equity funds[1], suffer from data biases, where the main biases are survivorship bias, selection bias, and backfill bias. Summing up, there is an overall positive bias that has to be taken into account (see [6]). Empirical studies, e.g. [5] and [27], illustrate the data biases in hedge fund returns, and account for it by subtracting an appropriate value from the hedge fund time series when replicating the returns.

Many empirical studies also found significant positive autocorrelation in hedge fund return series (see for example [17] and [10]). Positive autocorrelation means that today's returns depend on last periods' returns, and is created through market frictions like illiquidity, i.e. autocorrelation increases with market frictions. One way to consider autocorrelation when modeling hedge fund returns is to smooth the returns. Thereby, the reported or observed returns are a finite moving-average of unobserved economic returns (see [17]). With this model realistic levels of serial correlation can be generated for historical hedge-fund returns. The idea of using lagged returns for the explanation of reported returns is based on the empirical observation that in a market-model regression for hedge fund returns observable returns can be explained by a weighted average of the market's returns over the most recent periods (see [6]). Another way is direct modeling, as described in [17]. Among other things, he proposes to use a two-state Markov

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[1] For the dealing of selection bias in private equity funds, see, e.g., [13] and [29].

process.

Since the pioneering work of [19] Markov Switching Models (MSM) have become increasingly popular in economic studies (see, e.g., [9], [32] or [10]) and are able to capture non-normality and serial correlation. Furthermore, MSM can replicate more typical statistical properties of hedge funds, for example volatility clustering and the leverage effect.

Volatility clustering means that large changes in price tend to follow large changes in price, of either sign, and small changes tend to follow small changes. The leverage effect implies for stock markets that volatility is higher in a falling market than in a rising market.

In this article, we use an autoregressive two-state Markov Switching process to describe the evolution of hedge fund returns over time. We extend the existing MSM of [10] by deriving the autocorrelation function of the leverage effect for the method of moments according to [32]. To the authors' best knowledge, the leverage effect is not yet captured by applications of MSM in the literature. In contrast to [10] or [21] who use this model to derive optimal portfolios including alternative investments, we rather investigate the quality of the model in forecasting risk positions. Furthermore, we compare two different specifications of the parameter estimation method to analyse if it is worth to fit for the leverage effect that can be found in hedge fund data.

The main research questions we address in this article are the following: What are typical statistical properties of hedge fund indices and are there any differences for different regions? Is it worth including the leverage effect into the parameter estimation of a MSM used to predict the risk exposure or performance measures of hedge fund indices?

The article is organized as follows. Section 2 contains the statistical analysis of the hedge fund index data. In Section 3 we first describe the class of MSM in general and derive two specifications for the parameter estimation, which is then applied in Section 4. In Section 4 the results of the two compared MSM specifications subject to risk and performance measures are presented. Section 5 concludes.

2 Statistical Properties of Hedge Fund Indices

This section aims to characterize some of the typical features found in hedge fund index time series. Before we analyse our dataset, we first give a short summary on the statistical properties and the data used in other studies.

Compared to stocks and bonds, hedge funds reveal some typical statistical properties, which have been confirmed by a number of studies, for example [24]. Main statistical properties of hedge funds, as for example pointed out in [10], are:

- *Non-normality:* Hedge fund time series are characterized by negatively skewed and fat tailed returns. Skewness is defined as the degree of asymmetry of a probability distribution. A negative skewness implies that the left tail is the longest and that the mass of the distribution is concentrated on the right side of the density function. Kurtosis is defined as the fatness of the tails of a probability distribution. A normally distributed random variable has an excess kurtosis of zero. Now, a positive excess kurtosis implies fatter tails, meaning that extreme or tail events are more likely to occur.
- *Autocorrelation:* In contrast to long-only equity portfolios and mutual funds, hedge fund returns exhibit in most cases strong serial correlation (see, e.g., [17]. Positive serial correlation or autocorrelation means that today's return depends on last periods' returns. When today's return only depends on yesterday's return, we speak of first-order autocorrelation; when today's return depends on the return two (three) periods ago, we speak of second-order (third-order) autocorrelation.
- *Volatility Clustering:* Additionally, hedge fund time series often exhibit volatility clustering, which means that large changes in price tend to follow large changes in price, of either sign, and small changes tend to follow small changes. While the returns themselves may be uncorrelated, absolute returns or their squares can be positively autocorrelated. This means that volatility is dependent upon past realizations of the volatility process.

As already mentioned above, our special focus in this paper is on an additional property, the leverage effect (see, e.g., [2] for a theoretical description and [8] for an empirical study).

- *Leverage Effect:* The leverage effect implies that for stock markets volatility is higher in a falling market than in a rising market. The reason for this may be that when the equity price falls, the debt remains constant in the short term. So the debt-equity ratio increases, the firm becomes more leveraged, the future of the firm becomes more uncertain and the equity price therefore becomes more volatile. A simple estimator for the leverage effect is the empirical autocorrelation between current squared returns and the last period's returns.

Before we analyse if these features also apply to our data, we give a short summary on the data used in other empirical studies.

There are many studies that analyse hedge funds of exactly one region. For example, [18] and [31] focus on Asian hedge funds, [4] on European. Since most hedge funds are US-based, the results of most studies mainly refer to data from North America, see for example [27] and [11]. The latter also gives an overview on the main hedge fund data bases and their use in the academic literature.

In addition to empirical studies focusing on hedge funds or hedge fund indices from one specific region, there are – according to the authors' best knowledge – so far no studies that compare the properties of hedge funds from different regions.

One can only find, e.g., comparisons of the Asian and European stock market relating to the Asian Crisis 1997 (see [12]).

Therefore, the aim of this section is to compare hedge fund indices (HFI) from different markets. This is done by highlighting the special characteristics, equalities and differences regarding the different regional markets.

We use publicly available data from 'Eurekahedge', a hedge fund research company based in Singapore.[2] In the following we concentrate on the overall indices, i.e. including all different hedge fund styles, from North America, Europe, Asia, Emerging Markets, Eastern Europe & Russia, and Latin America.

The monthly log returns of those six HFI ranging from January 2000 to September 2008 are shown in Figure 1.

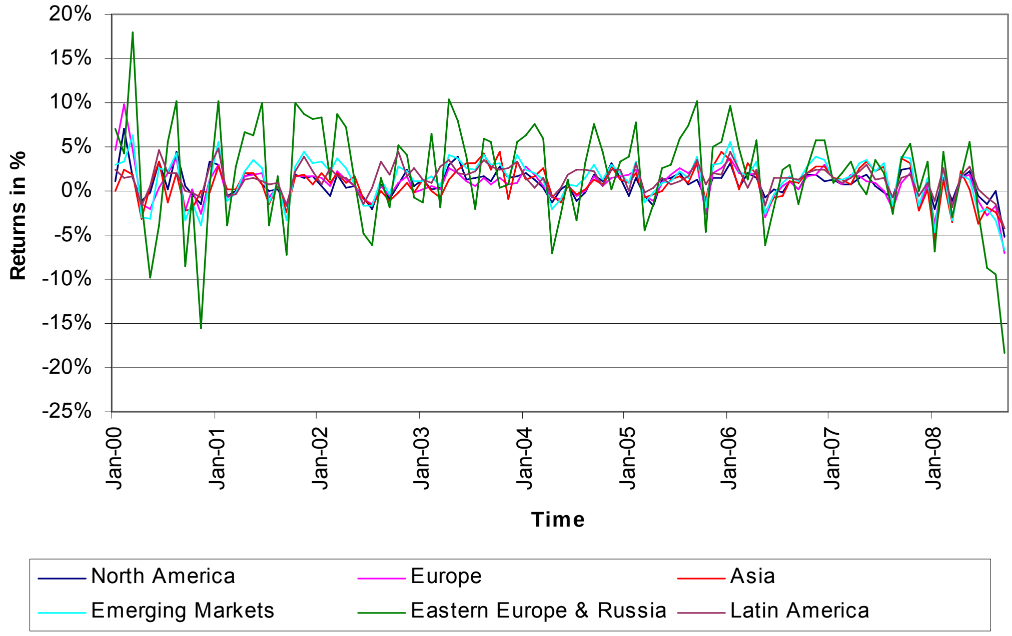

Figure 1: Monthly Log Returns of HFI

Table 1 gives a summary of the basic statistics for the six HFI. Most HFI - except for North America and Europe with a small positive skewness - exhibit a negative skewness. All HFI have a positive excess kurtosis implying fatter tails than for a normal distribution, again with the North American and European HFI differing from the other HFI by much higher values for the excess kurtosis. From the Jarque-Bera Test we see that the null hypothesis of normally distributed returns

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[2] For more information on the HFI see www.eurekahedge.com.

can be rejected for all HFI at a 5% significance level. As a result, we can say that the considered HFI are non-normally distributed.

	Basic Statistics						Jarque-Bera Test
	mean (in %)	std.dev. (in %)	skewness	excess kurtosis	max (in %)	min (in %)	p-value
North America	0.827	1.584	0.024	2.753	6.955	-5.274	0.000
Europe	0.766	1.995	0.096	4.666	9.794	-6.999	0.000
Asia	0.741	1.990	-0.596	0.178	4.417	-5.581	0.042
Emerging Markets	1.294	2.441	-0.797	0.328	6.322	-6.737	0.003
Eastern Europe	1.947	5.803	-0.673	1.134	17.917	-18.392	0.001
Latin America	1.405	1.544	-0.587	1.061	4.847	-4.299	0.004

Table 1: Basic Statistics and Jarque-Bera Test for HFI

The empirical autocorrelations, volatility clustering, and leverage effect are shown in Table 2. All HFI show a significant positive autocorrelation of lag 1. Volatility clustering, measured by the autocorrelation of squared returns, ranges from -0.044 to 0.286, whereas the leverage effect, measured by the autocorrelation of today's squared returns and last period's returns, ranges from -0.048 to 0.262. All three statistical properties of Table 2 can be statistically confirmed with the Ljung-Box Test for autocorrelations, and thus are in line with previous studies (see, e.g., [17]). Especially prominent are the high volatility clustering and leverage effect in the European HFI and the high leverage effect in Asian HFI.

One might wonder why the leverage effect of some of the HFI has a positive sign, whereas the leverage effect observed in stock markets is negative. This might be due to the fact that hedge funds can run different strategies, e.g. short selling, or invest in other assets than stocks. If we compute the leverage effect of the overall HFI for long-only strategies we observe a leverage effect of -0.101 which is in the line with the results known from stock markets.

To sum it up, our empirical HFI data reveal the typical features of hedge funds known from other studies: non-normality, serial correlation, volatility clustering, and leverage effect.

	autocorrelation of lag		volatility clustering	leverage effect
	1	2		
North America	0.096	-0.025	-0.044	-0.011
Europe	0.290	0.053	0.286	0.262
Asia	0.177	0.260	0.066	0.127
Emerging Markets	0.211	0.126	0.077	0.071
Eastern Europe	0.186	0.018	0.021	-0.048
Latin America	0.174	0.011	0.023	0.056

Table 2: Autocorrelations, Volatility Clustering and Leverage Effect of HFI

3 Markov Switching Model

3.1 Model Description

As hedge funds reveal some typical statistical properties like non-normality, autocorrelations, volatility clustering, and leverage effect, a normal distribution is not appropriate to describe the evolution of hedge fund returns. Instead, we use in the following a MSM as introduced in [19]. We extend the approach of [10] so that it is not only able to capture non-normality, autocorrelations, and volatility clustering, but also the leverage effect.

As time series processes can change dramatically over time, the idea of MSM is to model different states (or regimes) a time series process can be in. Each regime of a time series process is described by its own density function, which leads to the possibility of capturing typical features of hedge fund returns.[3] The changes of regimes are modelled via a Markov chain $(S_t)_{t\in\mathbb{N}}$ with transition probabilities given

[3] The joint density distribution reveals moments different from those of the single density functions, with especially the skewness and excess kurtosis being different from zero. Thus, the non-normality of hedge fund returns is accounted for, while for example the feature of volatility clustering is captured by time-varying volatilities.

by

$$P\left(S_t = j \mid S_{t-1} = i, S_{t-2} = k, \ldots\right) = P\left(S_t = j \mid S_{t-1} = i\right) = p_{ij},$$

(1)

where p_{ij} denotes the transition probability of changing from state i in period t-1 to state j in period t. The transition matrix is given by

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1N} \\ p_{21} & \ddots & & \vdots \\ \vdots & & & \\ p_{N1} & \cdots & & p_{NN} \end{bmatrix}.$$

(2)

In the following, we restrict ourselves to the case of N = 2 (as for N > 2 the number of parameters would become too high for a suitable parameter estimation), i.e. only two possible states, for a first-order Markov switching autoregressive time series process similar to [10].

The return R_t at time t is given by

$$R_t = \mu_{S_t} + \Phi \cdot \left(R_{t-1} - \mu_{S_{t-1}}\right) + \sigma_{S_t} \cdot \varepsilon_t,$$

(3)

where $|\Phi|<1$, $\varepsilon_t \sim N(0,1)$ i.i.d. ε_t and S_t are assumed to be independent at all leads and lags and R_t is stationary. $S_t = 1$ if the process is in state 1, and $S_t = 2$ if it is in state 2.

Let p_{11}, $p_{22} < 1$ and $p_{11} + p_{22} > 0$ so that the Markov chain is ergodic, then there exists a unique stationary distribution $\pi' = (\pi_1, \pi_2)$ given by

$$\pi_1 = \frac{p_{21}}{p_{12} + p_{21}} \quad \text{and} \quad \pi_2 = \frac{p_{12}}{p_{12} + p_{21}} \quad .$$

(4)

3.2 Estimation Procedure

The parameter vector

$$\theta = \left(\mu_1, \mu_2, \sigma_1, \sigma_2, \Phi, p_{12}, p_{21}\right)$$

(5)

can be estimated by moment matching. For this, we set up a system of seven equations to get an estimate for the seven parameters of θ.[4] We set up seven

[4] When the number of moment conditions is the same as the number of unknown parameters, the

equations by equating the first to fourth centred moments (four equations) to the corresponding distribution moments as well as the autocorrelation of lag 1 to its empirical counterpart (one equation). The remaining two equations can be chosen out of

- the autocorrelations of lag n, n > 1,
- the autocorrelation of squared returns (volatility clustering) of lag 1,
- and the autocorrelation between yesterday's returns and today's squared returns (leverage effect) of lag 1.

Again, the chosen autocorrelations are equated to the corresponding empirical autocorrelations.

The first to fourth centred moments and autocorrelations (serial correlation and volatility clustering) of a first-order Markov switching autoregressive process are given by (see [32] or [10]):

mean μ:

$$\mu = \pi' \mu_S \tag{6}$$

variance σ^2:

$$\sigma^2 = \pi' \left((\mu_S - \mu 1) \otimes (\mu_S - \mu 1) + \frac{\sigma_S^2}{1-\Phi^2} \right) \tag{7}$$

skewness s:

$$\begin{aligned} s = \tfrac{1}{\sigma^3} \Big[& \pi' \big((\mu_S - \mu 1) \otimes (\mu_S - \mu 1) \otimes (\mu_S - \mu 1) \big) \\ & + 3\Phi^2 \pi' \Big(\big(B (I_2 - \Phi^2 B)^{-1} \sigma_S^2 \big) \otimes (\mu_S - \mu 1) \Big) \\ & + 3\pi' \big((\mu_S - \mu 1) \otimes \sigma_S^2 \big) \Big] \end{aligned} \tag{8}$$

excess kurtosis ek:

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method of moments estimator equals the generalized method of moments (GMM) estimator, since the weighting matrix has no impact (see [19], Chapter 14).

$$\begin{aligned}ek = \tfrac{1}{\sigma^4}\Big[&\pi'\big((\mu_S-\mu 1)\otimes(\mu_S-\mu 1)\otimes(\mu_S-\mu 1)\otimes(\mu_S-\mu 1)\big)\\ &+6\pi'\big((\mu_S-\mu 1)\otimes(\mu_S-\mu 1)\otimes\sigma_S^2\big)\\ &+\pi'\left(I_2-\Phi^4B\right)^{-1}\left(3\sigma_S^4+6\Phi^2\left(B\left(I_2-\Phi^2B\right)^{-1}\sigma_S^2\right)\otimes\sigma_S^2\right)\\ &+6\Phi^2\pi'\left(\left(B\left(I_2-\Phi^2B\right)^{-1}\sigma_S^2\right)\otimes(\mu_S-\mu 1)\otimes(\mu_S-\mu 1)\right)\Big]-3\end{aligned}$$

(9)

autocorrelation of lag n ∈ ё autocor$_n$:

$$autocor_n = \frac{1}{\sigma^2}\left[\pi'\left(\left(B^n(\mu_S-\mu 1)\right)\otimes(\mu_S-\mu 1)\right)+\Phi^n\pi'\left(I_2-\Phi^2B\right)^{-1}\sigma_S^2\right] \qquad (10)$$

autocorrelation of squared returns of lag 1 autocorsq:

$$autocorsq = \frac{\mathrm{E}\left[\left(R_t^2-\mathrm{E}\left[R_t^2\right]\right)\left(R_{t-1}^2-\mathrm{E}\left[R_{t-1}^2\right]\right)\right]}{\mathrm{E}\left[\left(R_t^2-\mathrm{E}\left[R_t^2\right]\right)^2\right]} \qquad (11)$$

with

$$\begin{aligned}&\mathrm{E}\left[\left(R_t^2-\mathrm{E}\left[R_t^2\right]\right)\left(R_{t-1}^2-\mathrm{E}\left[R_{t-1}^2\right]\right)\right] =\\ &\quad=\Phi^2\pi'\left(\left(\left(I_2-\Phi^2B\right)^{-1}\sigma_S^2\right)\otimes\mu_S^2\right)\\ &\quad+\Phi^2\pi'\left(I_2-\Phi^4B\right)^{-1}\left(3\sigma_S^4+6\Phi^2\left(B\left(I_2-\Phi^2B\right)^{-1}\sigma_S^2\right)\otimes\sigma_S^2\right)\\ &\quad+\pi'\left(\left(B\left(I_2-\Phi^2B\right)^{-1}\sigma_S^2\right)\otimes\mu_S^2\right)+\pi'\left(\left(B\mu_S^2\right)\otimes\mu_S^2\right)\\ &\quad+\pi'\left(\left(B\left(I_2-\Phi^2B\right)^{-1}\sigma_S^2+B\mu_S^2\right)\otimes\sigma_S^2\right)\\ &\quad+4\Phi\pi'\left(\left(B\left(I_2-\Phi^2B\right)^{-1}\left(\sigma_S^2\otimes\mu_S\right)\right)\otimes\mu_S\right)-\left(\mathrm{E}\left[R_t^2\right]\right)^2\end{aligned}$$

and

$$\mathrm{E}\left[\left(R_t^2-\mathrm{E}\left[R_t^2\right]\right)^2\right]=$$

$$=\pi'\mu_S^4+6\Phi^2\pi'\left(\left(B\left(I_2-\Phi^2B\right)^{-1}\sigma_S^2\right)\otimes\mu_S^2\right)$$

$$+\pi'\left(I_2-\Phi^4B\right)^{-1}\left(3\sigma_S^4+6\Phi^2\left(B\left(I_2-\Phi^2B\right)^{-1}\sigma_S^2\right)\otimes\sigma_S^2\right)$$

$$+6\pi'\left(\mu_S^2\otimes\sigma_S^2\right)-\left(\mathrm{E}\left[R_t^2\right]\right)^2.$$

In our approach we also include the autocorrelation between yesterday's returns and today's squared returns of lag 1 (leverage effect) autocorle:

$$\text{autocorle}=\frac{\mathrm{E}\left[\left(R_t^2-\mathrm{E}\left[R_t^2\right]\right)\left(R_{t-1}-\mu\right)\right]}{\sqrt{\mathrm{E}\left[\left(R_t^2-\mathrm{E}\left[R_t^2\right]\right)^2\right]}\cdot\sigma} \tag{12}$$

with

$$\mathrm{E}\left[\left(R_t^2-\mathrm{E}\left[R_t^2\right]\right)\left(R_{t-1}-\mu\right)\right]=$$

$$=\pi'\left(\left(B\left(\mu_S-\mu 1\right)\right)\otimes\mu_S^2\right)+\pi'\Phi^2\left(B\left(I_2-\Phi^2B\right)^{-1}\left(\sigma_S^2\otimes\left(\mu_S-\mu 1\right)\right)\right)$$

$$+\pi'\left(\left(B\left(\mu_S-\mu 1\right)\right)\otimes\sigma_S^2\right)+2\pi'\Phi\left(\left(B\left(I_2-\Phi^2B\right)^{-1}\sigma_S^2\right)\otimes\mu_S\right),$$

where

$\pi=\begin{pmatrix}\pi_1\\ \pi_2\end{pmatrix}$, $\mu_S=\begin{pmatrix}\mu_1\\ \mu_2\end{pmatrix}$, $1=\begin{pmatrix}1\\ 1\end{pmatrix}$, $\sigma_S=\begin{pmatrix}\sigma_1\\ \sigma_2\end{pmatrix}$, $B=\begin{pmatrix}p_{11} & p_{12}\\ p_{21} & p_{22}\end{pmatrix}=\begin{pmatrix}1-p_{12} & p_{12}\\ p_{21} & 1-p_{21}\end{pmatrix}$,[5] I_2 is the two-dimensional identity matrix, Φ the autocorrelation parameter, R_t the return in period t, and $\otimes$ the element by element multiplication.

Hence, the parameter vector θ (see (5)) can be estimated via

$$\min_{\theta}\Big\{\left(\tilde{\mu}-\mu\right)^2+\left(\tilde{\sigma}-\sigma\right)^2+\left(\tilde{s}-s\right)^2+\left(\tilde{e}k-ek\right)^2$$

$$+\sum_{i=1}^{n}\left(\tilde{a}utocor_i-autocor_i\right)^2+1_{\{m=1\}}\cdot\left(\tilde{a}utocorsq-autocorsq\right)^2 \tag{13}$$

$$+1_{\{p=1\}}\cdot\left(\tilde{a}utocorle-autocorle\right)^2\Big\}$$

[5] The matrix B gives the transition probabilities for the "time-reversed" Markov chain that moves back in time. In the case with two states, the "backward" transition probability matrix B equals the "forward" transition probability matrix P. For more details see p.87 of [32].

with n + m + p = 3 and where $\tilde{\mu}$ resembles the sample mean, $\tilde{\sigma}^2$ the sample variance, $\tilde{s}$ the sample skewness, $\tilde{ek}$ the sample excess kurtosis, $\widetilde{autocor}_n$ ($n \in$ ë) the sample autocorrelations of lag n, $\widetilde{autocorsq}$ the sample autocorrelations of squared returns, and $\widetilde{autocorle}$ the sample autocorrelations of yesterday's returns and today's squared returns.

We tested different specifications of (13) and found the best results when using autocorrelation of lag 1, volatility clustering, and autocorrelation of lag 2 (see also [10]) or autocorrelation of lag 1, volatility clustering, and leverage effect. Hence, we will focus on these two variants in the empirical part.

4 Case Study

In this section we apply the model and estimation methods proposed in Section 3 to the HFI data from Section 2. In contrast to [10] or [21] we don't apply the model to an asset allocation problem, but test it in terms of another important application – risk management.

4.1 Parameter Estimation

The parameter estimates of the model from Equation (3) estimated via (13) with n = 2, m = 1, and p = 0 (method 1) as well as with n = 1, m = 1, and p = 1 (method 2) are given in Tables 3 and 4.

	North America	**Europe**	**Asia**	**Emerging Markets**	**Eastern Europe**	**Latin America**
μ_1	0.0082	0.0141	0.0082	0.0256	-0.0251	0.0073
μ_2	0.0079	0.0136	-0.0259	-0.0092	0.0288	-0.0060
σ_1	0.0221	0.0136	0.0129	0.0096	0.0739	0.0120
σ_2	0.0022	0.0017	0.0146	0.0223	0.0410	0.0212
Φ	0.0897	0.2781	-0.3061	0.4805	0.2012	0.1803
p_{12}	0.7006	0.0868	0.0763	0.4228	0.6680	0.2872
p_{21}	0.7170	0.0500	0.2726	0.6692	0.2824	0.6476

Table 3: Parameter Estimates for Method 1 (without autocorle)

	North America	Europe	Asia	Emerging Markets	Eastern Europe	Latin America
μ_1	0.0010	0.0136	0.0057	0.0235	-0.0278	0.0021
μ_2	0.0007	0.0139	0.0203	-0.0193	0.0391	0.0172
σ_1	0.0267	0.0012	0.0119	0.0139	0.0664	0.0165
σ_2	0.0042	0.0098	0.0072	0.0190	0.0424	0.0107
Φ	0.0952	0.2912	-0.2207	0.2378	0.0150	0.0145
p_{12}	0.6522	0.1821	0.0505	0.2012	0.2018	0.1110
p_{21}	0.6290	0.3391	0.0413	0.6074	0.0477	0.0230

Table 4: Parameter Estimates for Method 2 (with autocorle)

As one can easily see the parameter values and hence the properties of the models are quite different for the two methods. If we compare both methods in terms of absolute errors between theoretical moments implied by the estimated parameters and empirical moments (including the first four moments, autocorrelations of lag 1 and 2, volatility cluster, and leverage effect), we see that in most cases method 2 leads to a better fit in absolute terms, especially for those time series where we observed a high leverage effect (Europe and Asia).

	Method 1 (without autocorle)	Method 2 (with autocorle)
North America	0.0872	0.0610
Europe	0.0945	0.0538
Asia	0.3541	0.0801
Emerging Markets	0.0900	0.0813
Eastern Europe	0.1247	0.1258
Latin America	0.0817	0.1331

Table 5: Mean Absolute Deviations of Theoretical and Empirical Moments

4.2 Comparison of Risk and Performance Measures

As a direct measurement of the goodness of the models in terms of a test for residuals is not possible, we compare the two different variants of parameter estimation according to their impact on risk and performance measures in this section. The first variant (method 1) is similar to the one used e.g. in the asset allocation case study of [10], while the second one (method 2) takes the leverage effect into account instead of the autocorrelation of lag 2. Both of them take account for non-normality by including skewness and excess kurtosis and furthermore, the features volatility clustering and autocorrelation of lag 1 (see Table 6).

	Method 1	**Method 2**
non-normality	+	+
volatility clustering	+	+
autocorrelation lag 1	+	+
autocorrelation lag 2	+	
leverage effect		+

Table 6: Survey of MSM models and Incorporated Features

For both sets of estimated parameters, we simulate the monthly evolution of the HFI return time series for 105 months (the length of the historical time series), using a Monte Carlo Simulation with 10,000 paths. Then we compute the risk and performance measures value-at-risk (VaR), conditional value-at-risk (CVaR), adjusted Sharpe ratio (ASR), and Sharpe-Omega (SΩ) (for a definition of the measures see the Appendix) of the simulated monthly returns for each path and compare their averages with the empirical risk and performance measures. We concentrate on the 5% VaR and CVaR, respectively, because conclusions drawn from the 1% measures could be misleading as our empirical return series are rather small. We have chosen the adjusted Sharpe ratio instead of the more popular Sharpe ratio to account for the non-normality in HFI returns found in Section 2. As loss threshold L in the calculation of Sharpe-Omega we have chosen the risk-free rate.

Table 7 shows the empirical risk and performance measures as well as the average risk and performance measures of the simulated returns according to method 1 and 2, respectively. The last two columns additionally contain the mean absolute (MAE) and mean relative errors (MRE). As the adjusted Sharpe ratio is in absolute terms much higher than the other numbers, the comparison of mean absolute error might be misleading in some cases where the error in adjusted Sharpe ratio dominates the other terms.

		VaR	**CVaR**	**ASR**	**SΩ**		**MAE**	**MRE**
North America	Empirical	0.0241	0.0253	0.3594	0.0058			
	Method 1	0.0277	0.0297	0.3661	0.0057		0.0147	0.3478
	Method 2	0.0260	0.0261	0.1895	0.0031		0.1753	1.0432
Europe	Empirical	0.0331	0.0379	0.2562	0.0052			
	Method 1	0.0310	0.0355	0.7551	0.0053		0.5035	2.0998
	Method 2	0.0327	0.0369	0.3820	0.0053		0.1273	0.5571

Asia	Empirical	0.0334	0.0406	0.2407	0.0049		
	Method 1	0.0394	0.0384	0.1968	0.0038	0.0532	0.6480
	Method 2	0.0369	0.0386	0.2440	0.0049	0.0090	0.1767
Emerging Markets	Empirical	0.0461	0.0441	0.4023	0.0104		
	Method 1	0.0372	0.0315	0.5727	0.0123	0.1937	1.0788
	Method 2	0.0481	0.0458	0.3964	0.0102	0.0099	0.1215
Eastern Europe	Empirical	0.1059	0.1237	0.2816	0.0170		
	Method 1	0.1139	0.1347	0.2545	0.0157	0.0473	0.3362
	Method 2	0.1002	0.1091	0.3352	0.0178	0.0747	0.4135
Latin America	Empirical	0.0292	0.0222	0.6751	0.0116		
	Method 1	0.0304	0.0280	0.5816	0.0103	0.1017	0.5491
	Method 2	0.0267	0.0213	0.7419	0.0118	0.0705	0.2475

Table 7: Comparison of Simulated and Empirical Risk and Performance Measures

In most cases the MREs of method 2 are smaller than those of method 1. Only for the two regions that showed the lowest leverage effect in their HFI return series (see Table 2), North America and Eastern Europe, method 1 has a lower MRE. Especially for the two tail risk measures VaR and CVaR we observe in almost all cases better results for the model where we included the leverage effect in the fitting procedure than for the model without leverage effect in the objective function of the parameter estimation. Also for the performance measure Sharpe-Omega, which takes the whole distribution into account, the specification with leverage effect is clearly preferable, since it yields – except for the North American HFI – the smaller differences to the empirical performance measures. Only for the adjusted Sharpe ratio none of the two methods clearly dominates the other.

5 Conclusion

In our statistical examination we saw that there are typical features which occur in hedge fund index return series in (almost) all regions, e.g. non-normality, serial correlation, volatility clustering, and leverage effect. Nevertheless, there are regional differences, e.g. the North American index shows the lowest autocorrelation and only very small volatility clustering and leverage effect, whereas the European index and the Asian index have much higher autocorrelations, volatility clusters, and leverage effect.

As already pointed out in the introduction, a sound risk management for hedge funds is essential to anticipate the risk exposure. Therefore, we extended the MSM model of [10] by including the leverage effect, which is a typical hedge fund feature with impact on the risk exposure, in the parameter estimation. Then we compared the two MSM specifications (one with leverage effect (method 2) the other without (method 1)) on a set of hedge fund indices from different regions to see which specification is better in forecasting risk and performance measures.

In a case study we showed that it is worth including the leverage effect in the parameter estimation via moment matching if there is a leverage effect in the data. Using this method the goodness of risk and performance measures could be increased significantly.

Appendix

A Proof of Markov Regime Switching Formula

The proofs for the first four centred moments, autocorrelation of lag n, and autocorrelation of squared returns of lag 1 can be found in [32]. For sake of completeness we derive here the formula of the leverage effect (Equation (12)). This is similar to the derivation of the autocorrelation formulas (see, e.g., [32]).

The underlying process is given by Equation (3). Further assumptions are:

1. The process starts from its steady-state distribution.
2. The error terms ε_t are iid.
3. ε_t and S_t are independent at all leads and lags.
4. R_t is stationary.

Using (3) we get for the squared return R_t^2 at period t

$$\begin{aligned} R_t^2 &= \left(\mu_{S_t} + \Phi\left(R_{t-1} - \mu_{S_{t-1}}\right) + \sigma_{S_t}\varepsilon_t\right)^2 \\ &= \mu_{S_t}^2 + \Phi^2\left(R_{t-1} - \mu_{S_{t-1}}\right)^2 + \sigma_{S_t}^2\varepsilon_t^2 + 2\mu_{S_t}\Phi\left(R_{t-1} - \mu_{S_{t-1}}\right) \\ &+ 2\mu_{S_t}\sigma_{S_t}\varepsilon_t + 2\Phi\left(R_{t-1} - \mu_{S_{t-1}}\right)\sigma_{S_t}\varepsilon_t \end{aligned} \tag{14}$$

Noting, that linear terms of ε_t will be uncorrelated with terms dated period t or earlier, the last line of (14) can be omitted when inserting (14) into the following equation:

$$
\begin{aligned}
&E\left[\left(R_t^2 - E\left[R_t^2\right]\right)\left(R_{t-1}-\mu\right)\right] = \\
&= E\left[\left(\mu_{S_t}^2 + \Phi^2\left(R_{t-1}-\mu_{S_{t-1}}\right)^2 + \sigma_{S_t}^2\varepsilon_t^2 + 2\mu_{S_t}\Phi\left(R_{t-1}-\mu_{S_{t-1}}\right) - E\left[R_t^2\right]\right)\cdot\right. \\
&\left.\cdot\left(\left(R_{t-1}-\mu_{S_{t-1}}\right)+\left(\mu_{S_{t-1}}-\mu\right)\right)\right] \\
&= E\left[\mu_{S_t}^2\left(R_{t-1}-\mu_{S_{t-1}}\right)\right] + E\left[\mu_{S_t}^2\left(\mu_{S_{t-1}}-\mu\right)\right] + E\left[\Phi^2\left(R_{t-1}-\mu_{S_{t-1}}\right)^3\right] \\
&+ E\left[\Phi^2\left(R_{t-1}-\mu_{S_{t-1}}\right)^2\left(\mu_{S_{t-1}}-\mu\right)\right] + E\left[\sigma_{S_t}^2\varepsilon_t^2\left(R_{t-1}-\mu_{S_{t-1}}\right)\right] \\
&+ E\left[\sigma_{S_t}^2\varepsilon_t^2\left(\mu_{S_{t-1}}-\mu\right)\right] + E\left[2\mu_{S_t}\Phi\left(R_{t-1}-\mu_{S_{t-1}}\right)^2\right] \\
&+ E\left[2\mu_{S_t}\Phi\left(R_{t-1}-\mu_{S_{t-1}}\right)\left(\mu_{S_{t-1}}-\mu\right)\right] \\
&- E\left[E\left[R_t^2\right]\left(R_{t-1}-\mu_{S_{t-1}}\right)\right] - E\left[E\left[R_t^2\right]\left(\mu_{S_{t-1}}-\mu\right)\right]
\end{aligned}
\tag{15}
$$

where all covariance terms that include ε_t, S_t or $\left(R_t - \mu_{S_t}\right)$ with odd exponents cancel out. This follows from assumptions 2 and 3. The last two terms of (15) can also be deleted, due to the fact that $\mu = E\left[R_t\right] = \pi'\mu_S$ (for a proof see [32]).

Hence, (15) reduces to

$$
\begin{aligned}
&E\left[\left(R_t^2 - E\left[R_t^2\right]\right)\left(R_{t-1}-\mu\right)\right] = \\
&= \underbrace{E\left[\mu_{S_t}^2\left(\mu_{S_{t-1}}-\mu\right)\right]}_{T1} + \underbrace{E\left[\Phi^2\left(R_{t-1}-\mu_{S_{t-1}}\right)^2\left(\mu_{S_{t-1}}-\mu\right)\right]}_{T2} \\
&+ \underbrace{E\left[\sigma_{S_t}^2\varepsilon_t^2\left(\mu_{S_{t-1}}-\mu\right)\right]}_{T3} + \underbrace{E\left[2\mu_{S_t}\Phi\left(R_{t-1}-\mu_{S_{t-1}}\right)^2\right]}_{T4},
\end{aligned}
\tag{16}
$$

where the terms T1 and T3 are given by

$$T1 = \pi'\left(\left(B\left(\mu_S - \mu 1\right)\right)\otimes\mu_S^2\right)$$

(17)

and

$$T3 = \pi'\left(\left(B\left(\mu_S - \mu 1\right)\right)\otimes\sigma_S^2\right).$$

(18)

To derive expressions for the second and forth terms T2 and T4, notice that

$$
\begin{aligned}
E\left[\left(R_{t-1}-\mu_{S_{t-1}}\right)^2 \mid S_t\right] &= B\cdot E\left[\left(R_{t-1}-\mu_{S_{t-1}}\right)^2 \mid S_{t-1}\right] \\
&= B\cdot\sum_{i=0}^{\infty}\Phi^{2i}B^i\sigma_S^2 \qquad (19)\\
&= B\left(I_2-\Phi^2B\right)^{-1}\sigma_S^2,
\end{aligned}
$$

where I_2 is the 2-dimensional identity matrix and B the backward transition probability matrix.[6] The third equation follows from the property of a geometric series and the fact that $(I_2 - \Phi^2B)$ is invertible. Since $|\Phi| < 1$ this will automatically be satisfied for all transition probability matrices since B has a single eigenvalue equal to one and its remaining eigenvalue is smaller than one.

Using (19), we get

$$
T2 = \pi'\Phi^2\left(B\left(I_2-\Phi^2B\right)^{-1}\left(\sigma_S^2\otimes\left(\mu_S-\mu 1\right)\right)\right) \qquad (20)
$$

and

$$
T4 = 2\pi'\Phi\left(\left(B\left(I_2-\Phi^2B\right)^{-1}\sigma_S^2\right)\otimes\mu_S\right).
$$

(21)

Inserting (17), (18), (20) and (21) into (16), we receive the nominator of the leverage formula (12).

To derive the denominator of (12), we use the variance formula

$$
\sigma^2 = E\left[\left(R_t-\mu\right)^2\right] = \pi'\left(\left(\mu_S-\mu 1\right)\otimes\left(\mu_S-\mu 1\right)+\frac{\sigma_S^2}{1-\Phi^2}\right), \qquad (22)
$$

and the formula for the fourth centred moment (for a proof see [32])

$$
\begin{aligned}
E\left[\left(R_t-\mu\right)^4\right] &= \pi'\left(\left(\mu_S-\mu 1\right)\otimes\left(\mu_S-\mu 1\right)\otimes\left(\mu_S-\mu 1\right)\otimes\left(\mu_S-\mu 1\right)\right) \\
&+ 6\pi'\left(\left(\mu_S-\mu 1\right)\otimes\left(\mu_S-\mu 1\right)\otimes\sigma_S^2\right) \\
&+ \pi'\left(I_2-\Phi^4B\right)^{-1}\left(3\sigma_S^4+6\Phi^2\left(B\left(I_2-\Phi^2B\right)^{-1}\sigma_S^2\right)\otimes\sigma_S^2\right) \\
&+ 6\Phi^2\pi'\left(\left(B\left(I_2-\Phi^2B\right)^{-1}\sigma_S^2\right)\otimes\left(\mu_S-\mu 1\right)\otimes\left(\mu_S-\mu 1\right)\right) .
\end{aligned}
$$

Setting $\mu = 0$ we obtain

---

[6] For more details see Footnote 5 or p.87 of [32].

$$
\begin{aligned}
E\left[\left(R_t^2 - E\left[R_t^2\right]\right)^2\right] &= E\left[R_t^4\right] - \left(E\left[R_t^2\right]\right)^2 \\
&= \pi' \mu_S^4 + 6\pi'\left(\mu_S^2 \otimes \sigma_S^2\right) \\
&+ \pi'\left(I_2 - \Phi^4 B\right)^{-1}\left(3\sigma_S^4 + 6\Phi^2\left(B\left(I_2 - \Phi^2 B\right)^{-1}\sigma_S^2\right)\otimes \sigma_S^2\right) \\
&+ 6\Phi^2\pi'\left(\left(B\left(I_2 - \Phi^2 B\right)^{-1}\sigma_S^2\right)\otimes \mu_S^2\right) - \left(E\left[R_t^2\right]\right)^2
\end{aligned}
\tag{23}
$$

which completes the proof.

B Risk and Performance Measures

In this section, we give an overview on some well-known risk and performance measures. We differentiate between tail related risk measures like value-at-risk and conditional value-at-risk and performance measures based on the whole distribution like Sharpe ratio, adjusted Sharpe ratio, Omega, and Sharpe-Omega.

Value-at-Risk and Conditional Value-at-Risk

The value-at-risk VaR_α of a random variable R to a confidence level 1 - α is defined by (see, e.g., p. 252 of [34])

$$VaR_\alpha(R) = E(R) - R_\alpha,$$

(24)

where

$$R_\alpha = \sup\{x \in R : P(R < x) \le \alpha\}$$

(25)

is the value of R that will be exceeded with probability 1 - α.

Despite its popularity, VaR has some negative properties: it does not address the distribution of potential losses on those rare events when the VaR estimate VaR_α is exceeded. Furthermore, it is not coherent (see, e.g., p. 254 of [34]). A risk measure that addresses both disadvantages is the conditional value-at-risk.

The conditional value-at-risk (CVaR) is a risk measure that focuses on the losses which exceed VaR. In the literature, CVaR is also sometimes referred to as Expected Shortfall.[7]

$CVaR_\alpha$ to a predefined confidence level 1 - α is defined as the average loss given

[7] Note that this is only true for continuous random variables. For more details and proof see [1], p. 10.

that R_α is exceeded (see, e.g., p. 263 of [34]), i.e.

$$\mathrm{CVaR}_\alpha(R) = -\mathrm{E}(R \mid R \leq R_\alpha),$$

(26)

where R_α is given by (25).

As already mentioned, CVaR is a coherent risk measure (see, e.g., [1]) and therefore more appropriate than VaR when assessing the risk of a portfolio.

Sharpe Ratio and Adjusted Sharpe Ratio

A popular performance measure is the Sharpe ratio (SR) introduced by William F. Sharpe in 1966. It measures the risk-adjusted performance of an investment or a trading strategy relative to a benchmark asset, such as the risk-free rate of return.

The SR of an asset return R, SR(R), is defined as the expected excess return per unit of risk associated with the excess return, i.e.

$$\mathrm{SR}(R) = \frac{\mathrm{E}(R) - r_f}{\mathrm{STD}(R)} = \frac{\mu_R - r_f}{\sigma_R},$$

(27)

where the expected excess return is given as the expected asset return Æ(R) = μ_R beyond the risk free rate of return r_f and the risk is given by the standard deviation of R, STD(R) = σ_R.

The SR only is an appropriate performance measure when the return distribution solely depends on two parameters: location and scale parameter. Thus the SR does not measure correctly the performance of non-normally tailed or skewed return distributions, such as those of fat tailed and negatively skewed hedge fund returns. Therefore, a more generalized SR, called the adjusted Sharpe ratio, is presented.

The adjusted Sharpe ratio (ASR) extends the SR by taking non-normality in form of skewness and excess kurtosis of the asset returns into account. The ASR of an asset return R, ASR(R), is given by (see, e.g., p. 44 of [3])

$$\mathrm{ASR}(R) = \mathrm{SR}(R) \cdot \left[1 + \frac{s(R)}{6}\mathrm{SR}(R) - \frac{ek(R)}{24}\mathrm{SR}(R)^2\right],$$

(28)

where s(R) and ek(R) represent the skewness and excess kurtosis of an asset return R, respectively.

Although the ASR is more appropriate than the SR for measuring the performance of non-normally distributed asset returns, the ASR still does not capture all the information contained in the return series – basically it only captures the information contained in the first four moments.

Omega and Sharpe-Omega

The Omega measure introduced by [26] was developed with the intention to take the entire return distribution into account and is defined as the ratio of the gain with respect to a threshold L and the loss with respect to the same threshold. Then the Omega measure Ω_L with respect to a threshold L is given by [8]

$$\Omega_L = \frac{\int_L^b \left[1 - F(r)\right] dr}{\int_a^L F(r)\, dr}$$

(29)

where (a,b) is the support of the return distribution and F is the cumulative distribution of returns.

A new version of Omega introduced by [25] is the so called Sharpe-Omega given by

$$S\Omega_L = \frac{E(R) - L}{\int_a^L F(r)\, dr} = \Omega_L - 1.$$

(30)

The formula for Sharpe-Omega consists of the numerator of (27), where the risk-free rate of return r_f is replaced by the threshold L, and the denominator of (29).

Since the numerator of (30) corresponds to the price of a put option, Sharpe-Omega represents a measure of risk/return that is more intuitive than Omega. Since the price of the put option is the cost of protecting an investment's return below the target ratio (given by L), it is a reasonable measure of the investment's riskiness.

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