Procurement via Continuous-Time Multi-Attribute Double Auctions

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Abstract

Many trades in public and corporate procurement require an agreement on multiple attributes—besides the price, there are additional attributes describing product characteristics and the nature of the contract. The question is, how a market mechanism can provide support for the utilization of integrative gains and, simultaneously, keep search costs as low as possible. To this end, the paper outlines the working of a centralized multi-attribute continuous-time double auction mechanism.

1 Introduction

Oftentimes, procurement processes not only involve buying a pre-specified good at a price listed by the seller. Instead, there are additional attributes besides the price. These attribute further detail product characteristics—like for example size, weight, and material—and the nature of the contract—like for example delivery time, return policies, payment terms, as well as warranty terms and conditions.

Posted offer markets do not adequately account for the integrative potential of these multi-attribute scenarios. Bilateral negotiations, on the contrary, support the identification of Pareto optimal solutions. However, they are relatively time-consuming and costly, especially, if one conducts negotiations on the same goods with several partners at a time. This shortcoming is lessened but not neutralized by means of negotiation support systems (e.g. [4, 12]).

Multi-bilateral negotiations and multi-attribute auctions facilitate simultaneous search for agreements with more than one potential partner. A township, for example, can run a reverse auction to procure new computers. However, the manufactures have to be pointed at the negotiation and they have to (manually) bid in many auctions run by different townships and corporate buyers. There is a good deal of literature on single-sided multi-attribute auctions, but up to now the question how to coordinate several auctions on similar items is hardly addressed.

The paper outlines a multi-attribute double auction mechanism [6]. Conventional double auctions deal with a single attribute only; usually with the price. By adding the functionality to handle multi-attribute orders, this popular class of mechanisms used by virtually all financial exchanges becomes applicable in a wide variety of procurement scenarios.

The remainder of the paper is structured as follows: Section 2 briefly outlines requirements for a multi-attribute double auction mechanism. Section 3 describes the working of conventional single-attribute double auctions, before Section 4 extends the ideas to multi-attribute cases. Section 5 reviews related work and, finally, Section 6 concludes and outlines possible future work on the topic.

2 Desirable Properties

A multi-attribute double auction should possess the following properties from an axiomatic mechanism design viewpoint:

1. The mechanism should be *individually rational* meaning that agents prefer participating in the mechanism to trading via an outside venue. If the mechanism would not be individually rational, agents would refrain from participation thereby rendering the mechanism useless.

2. An agreement or trade is *Pareto optimal* if no other trade would make at least one of the respective agents better off without worsening the situation for any other agent. Pareto optimality is desirable, as agents could otherwise be better off with finding agreements outside of the mechanism.

3. The mechanism should be *coalition-proof*, i.e. there is no subset of agents that can benefit from colluding and trading outside the mechanism. As the two previous properties, this aims at encouraging participation in the mechanism.

4. The mechanism should be *budget-balanced*. In the weak form this requires that the market operator does not have to subsidize trading. In the strong form it even requires that trading is a zero-sum game. However, if none of the two forms of budget balance would be given, the operator would run the mechanism at a loss.

3 Single-Attribute Double Auctions

The working of a single-attribute double auction is outlined to introduce the concepts of matching and arbitration which will later on be used in Section 4 to describe the multi-attribute double auction.

"In a double auction, both buyers and sellers can submit bids, (offers to buy) and asks (offers to sell)" [5]. Orders or bids in a conventional double-auction are price-quantity-pairs submitted to the electronic auction system. Upon arrival of an order, the system checks whether it can immediately be matched against orders waiting on the other side of the market; if so, it is executed instantaneously. If the order cannot be executed, it is put in an order book and waits for being matched against subsequent orders.

In a single-attribute continuous-time double auction with limit orders, there is a bid-ask spread and never more than one order intersecting with orders on the other side of the market. Figure 1 displays a potential order book with three bids (B1 to B3) and three asks (A1 to A3). Each order is characterized by its market side (bid or ask) and a limit price.[1] The agent who submitted A3, for example, is willing to sell at a price of 100 or above; for A2 it is 103 or above and so on.

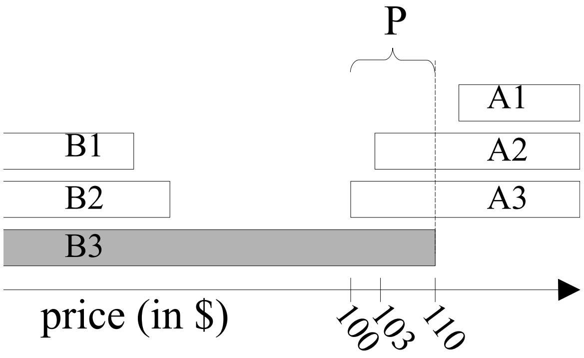

Figure 1: Single-attribute order book

Participants' preferences over the price are monotonous: a seller would always accept a price higher than the one he posted in his order and a buyer would always prefer a lower price. The system implies this and actually treats an order price p as an interval of acceptable prices ranging from zero to p, or from p to infinity. This idea can be extended to the multi-attribute case—an order is not a single point in the attribute-space, but a subspace. The direction of monotonicity of preferences on each attribute has either to be known implicitly, or to be explicitly specified.

At each time, at most one order intersects with the other market side. In Figure 1 this is bid B3. If one omits B3, there is a bid-ask spread in between the highest bid (here B2) and the lowest ask (here A3)—no trade can be made at this time. If order B3 enters the auction, an intersection occurs, a trade can be executed immediately, and the intersection disappears again.

Matching is the process of selecting a single order which intersects with the newly entered order. With the orders displayed in Figure 1, B3 can be matched with either A2 or A3. A common matching rule in financial markets is to take the ask with the lowest or the bid with the highest limit respectively. If two or more orders specify the same limit price, a tie-breaking rule like the earlier order entry time is applied. This is the so called price-time-preference. In the above example, B3 is matched with A3, the ask with the lowest limit price.

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[1] The order volume is irrelevant in this context and therefore omitted.

Arbitration is the process of determining the exact specification of the trade among two matched orders. Oftentimes, the limits of a matched pair do not coincide and there is more than one possible trade specification. The interval P gives the potential prices. For orders A3 and B3, any price $p \in P = [100, 110]$ is individual rational as well as Pareto optimal and budget-balanced and can therefore be chosen. Common arbitration rules are to take either the limit of the order in the order book ($p = 100$), the incoming order's limit ($p = 110$), or the midpoint of the arbitration interval ($p = 105$) [16]. However, prices $p > 103$ are not coalition-proof; the agents who submitted B3 and A2 would both be better off forming a coalition outside the auction mechanism and trading for a price of, for example, 103. Therefore, the requirements from section 2 call for any price $p \in [100, 103]$—which exact price is *fair*, *just*, or *best* cannot be answered unanimously.

The presented concepts are now generalized to the multi-attribute case.

4 Multi-Attribute Double Auctions

Two key concepts introduced in the last section are (1) matching, and (2) arbitration. Matching refers to testing whether an incoming order is compatible with any order in the order book and, if it is compatible with several orders, to select a single one among them. This process bases on hard constraints on what is an acceptable deal for an agent submitting an order and what is not. Arbitration refers to defining a specific agreement for two orders, once they are matched. Arbitration draws on concepts from cooperative game theory and requires, that the hard constraints on the acceptability are accompanied by soft constraints on the desirability of different possible trades. These soft constraints are expressed as utility functions in the auction.

4.1 Matching Orders

In the single-attribute case the incoming order was matched with the one potentially offering the highest utility. More specifically, the selling order with the lowest price was selected. Analogously, the bid which potentially[2] offers the highest utility to the incoming order is chosen.

Figure 2 sketches a possible order space in an Edgeworth box. For the sake of simplicity the example and the graphical representation are limited to a two-attribute scenario. However, the mechanism is equally applicable for more than two attributes. In this example, buyers and sellers submit offers for trading computers; the only two negotiable attributes are the price and the hard disk capacity.[3]

In Figure 2 there are two orders to buy a computer (B1 and B2) and one order to sell a

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[2] *Potentially* refers to the fact that the arbitration might select a trade specification which gives a lower utility than would have been possible when matched with another order. However, in this case, the arbitration rule would not be coalition-proof.

[3] Note that in a multi-attribute scenario it is not straightforward to use the terms *buyer* and *seller*. However, they are used in the following without restricting the generalizability as the mechanism does not depend on the wording.

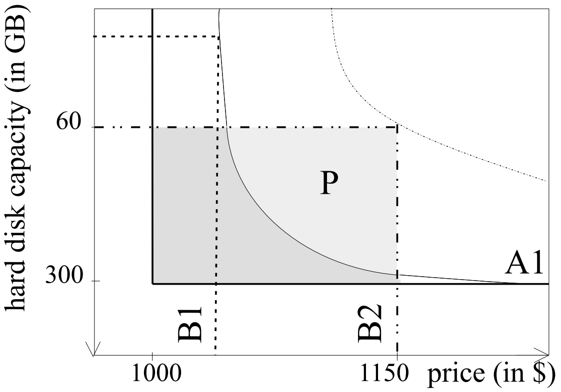

Figure 2: Edgeworth box exemplifying the matching of orders

computer (A1). Each order represents a subspace of the space of possible agreements. Order A1, for example, indicates that the respective agent is willing to sell any computer with a hard disk capacity not greater than 300 and at a price not smaller than 1000.[4] Obviously, not each point in this subspace of the agreement space is equally preferable for the agent. In the example, he prefers points in the upper right direction of the space. The direction of monotonicity has to be indicated in an offer. A higher price and a lower capacity are better for the seller; this is indicated by the two indifference curves shown.

Order A1 intersects with both buy orders B1 and B2. Therefore, the seller could basically sell to both buyers. The matching process has to select a single buy order to be matched with the sell order. As the buyers face competition and seller has some bargaining power, the buy order which is more preferable for the seller is chosen. In this example, it is offer B2 giving the seller a higher utility.

4.2 Arbitrating Among Orders

As in the single-attribute case, matching selected a pair of orders, and the arbitration rule has to determine the exact specification for the trade. More specifically, the matching step selected the order pair A1/B2 and now the arbitration step has to fix a specific point in the agreement space, i.e. a combination of a hard disk capacity and a price.

The final trade can basically be any point in the intersection of the two matched orders, i.e. the union of the dark grey and the light grey area depicted in Figure 2. Trading any computer with a hard disk capacity in between 60 and 300 at a price between 1000 and 1150 is individual rational for both agents as well as budget-balanced. However, most of these trades are not coalition-proof. One has to keep in mind that A1 intersects with B1 as well. Therefore, the dark grey region from Figure 2 is not coalition-proof. If B2 would request a deal with a hard disk capacity of 100 at a price of 1000, for example, the agent who submitted A1 could be better off if matched with B1. If the mechanism would impose any deal within the dark grey

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[4] Note the downward orientation of the y-axis.

area, the seller would be better off to reach an agreement outside the mechanism and might refrain from participation.

All trades in the light grey area—denoted P in congruence with the interval P shown in Figure 1—are not only individually rational and budget-balanced but they are coalition-proof as well. The final step in arbitration is to select a Pareto optimal point within this area.

The area P from Figure 2 can be mapped to the utility space in Figure 3. The mechanism should restrict its choice to Pareto optimal solutions, i.e. the upper right boundary of the possible utility space. Which point to choose on this Pareto optimal line is, as in the single-attribute case, not unambiguous.

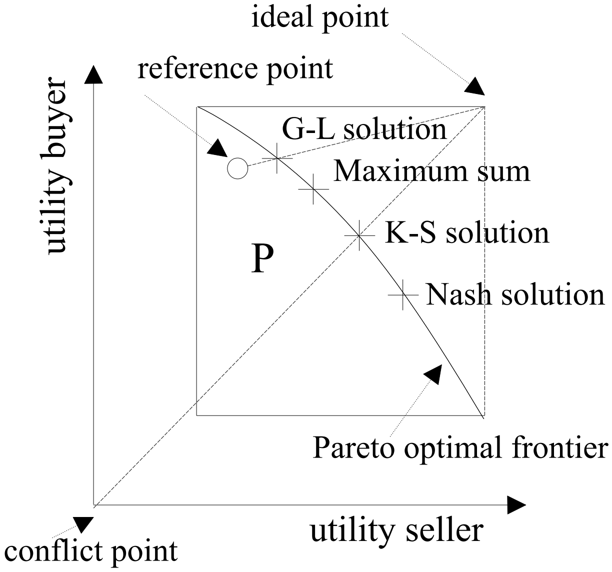

Figure 3: Arbitration in the utility space

The mechanism has the role of an arbitrator in a bilateral multi-issue negotiation. Cooperative game theory provides several arbitration rules, each of which aims at finding a *fair* or *just* solution. These arbitration rules require cardinal utility while up to now ordinal utility was sufficient. The interpersonal utility comparisons require rather strict assumptions on agents' stated utility—see [9] for a discussion of this matter. It is assumed that the agents submit utility functions along with their offers.

The Nash bargaining solution [14], and the Kalai-Smorodinsky bargaining solution [10] are two possible concepts for arbitrating the bilateral multi-issue negotiation. The Nash bargaining solution bases on several axioms which supposedly lead to a fair solution and specifies a unique solution conforming with the axioms. One of the axioms is Pareto optimality; this is the reason why the Nash solution is appropriate for selecting a Pareto optimal point in the space P of possible agreements among the matched offers. See [14] for the other axioms.

For identifying the Nash solution for the example displayed in Figure 3 one needs the concept of a conflict point, i.e. the outcome for both agents if they would not reach an agreement. The conflict point can be interpreted as no trade at all and can be assumed to yield zero utility to both players. The Nash solution is the trade which maximizes the product of both agents difference between the utility gained by the trade and the utility arising from the conflict outcome. If the conflict outcome gives zero utility to both agents, the Nash solution is simply the trade maximizing the product of agents' utilities. It is displayed in Figure 3.

Kalai and Somorodinsky proposed a slightly different axiomatisation [10]. While the Nash solution bases on the conflict point, the Kalai-Smorodinsky solution additionally requires an ideal point defined as the point which would give both agents the highest utility they could expect from any solution. This ideal point is usually not achievable; sometimes it is called *utopia*. The Kalai-Smorodinsky solution is defined as the point at which the straight line from the conflict outcome to the ideal outcome intersects the curve of Pareto optimal solutions. The Kalai-Smorodinsky solution gives each agent the same share of the best outcome he could possibly expect. Again, it is sketched in Figure 3.

Gupta and Livne propose an axiomatic model which introduces another prominent point termed reference point [7]. It is an extension of the Kalai-Smorodinsky solution in the sense that the solutions coincide if the conflict point is the reference outcome. However Gupta and Livne argue that oftentimes there is a reference point different from the conflict point. The reference point could, for example, be the outcome if both agents meet half way on each single attribute. The solution proposed by Gupta and Livne is to choose a trade specification which gives each agent the utility on the intersection of the Pareto optimal frontier and the straight line from the reference point to the ideal point. One such reference-point based solution is sketched in Figure 3.

The fourth reasonable solution displayed in Figure 3 is to maximize the sum of utilities (cf. [8]). All these solutions to the bilateral bargaining problem can be utilized as arbitration rule in a multi-attribute double auction. Furthermore, a plethora of other arbitration rules is possible as well. Unfortunately, there is no *best* solution or the one and only *fair* way to arbitrate among matched offers. Any of the above rules has its virtues and the specific choice depends on which of them most closely resembles the market engineers needs.

How does the choice of a specific arbitration rule influence incentive compatibility? Achieving incentive compatibility, i.e. giving the participants an incentive to truthfully reveal their preferences, is impossible with any mechanism meeting the requirements from Section 2 [13]. However, the degree of approximation might very well be different for different arbitration rules. The analysis of this interrelation is subject to future research.

5 Related Work

The work most closely related to the present paper is the multi-attribute trading of financial derivatives presented by [6] and the combinatorial double auction for trading grid resources outlined by [17]. Besides this, many single-attribute auction institutions can be extended to multi-attribute scenarios and the performance—e.g. allocative efficiency or maximization of a single participants' utility—can be studied. There are mainly four branches of research on multi-attribute auctions: (1) preference elicitation, (2) preference revelation, (3) bidding languages, and (4) performance measurements.

Preference elicitation deals with helping an agent to externalizes his preferences and tradeoffs across multiple attributes; an example is given in [1]. It is more of a decision-theoretic problem contributing to multi-attribute auctions, than a pure auction-theoretic problem. The usual decision theoretic techniques can be applied. See for example [21] and [11] for an overview on preference elicitation.

Studies on preference revelation are concerned with the question whether an agent, e.g. a single buyer running a procurement auction, should reveal its preferences over different configurations, for example by publishing an utility function. The effects of preference revelation on the performance of different auction mechanisms can be studied; [18] is an example for this. Up to now, there is no study of the effect of preference revelation in multi-attribute double auctions.

Once an agent elicited his preferences, he has to submit them via a bidding language to the auction mechanism. The simplest version is to submit several attribute-value pairs. However, a more sophisticated bidding language, like the configurable multi-attribute orders employed in [2] or the languages presented by [15], can allow for agents submitting complex orders with several options for attribute values and dependencies across attributes. Thereby, an extensive enumeration of configurations can be avoided.

Furthermore, there is a substantial literature on one-sided multi-attribute auctions and their performance, e.g. [3, 20]. The comparison of a multi-attribute English and a multi-attribute Vickrey auction, for example, is presented in [19] with a focus on agents' utilities and allocative efficiency.

All four objects of research on single-sided multi-attribute auctions relate to multi-attribute double auctions as well. Preference elicitation techniques are a client-side support which can directly be transferred to double auctions. The effect of the design alternatives outlined in Section 4 on an auction's performance will be subject to future research as well as different bidding languages and the effect of preference revelation.

6 Conclusion and Future Work

The paper outlined multi-attribute continuous-time double auctions. By utilizing the presented class of mechanisms, costs for searching trading partners and negotiating multi-attribute deals can be reduced. The presented auction is developed with respect to four major requirements: individually rationality, Pareto optimality, coalition proofness, and budget-balance. To achieve these properties the concepts of matching and arbitration are extended from well-known single-attribute double auction institutions to the presented multi-attribute double auction.

The interrelation of specific arbitration rules and the strategic behavior of agents will be subject to future research. Incentive compatibility cannot be achieved in any such mechanism, as mentioned earlier, but a question will be in how far incentive compatibility can be approximated. Laboratory experiments and computer-based simulations as well as other tools from the market engineering toolbox will be used for this matter. Furthermore, the effect of preference revelation on the agents' strategies and the market outcome will be studied.

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