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Formalization of Financial Problems and Solutions under Risk as an Essential Requirement for IT-based Financial Planning

by

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Formalization of Financial Problems and Solutions under Risk as an Essential Requirement for IT-based Financial Planning

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Abstract

At the beginning of the new millennium, the financial services market is experiencing a fundamental shift. More transparent and global markets through new means of communication on the one hand and better informed and more demanding customers on the other hand have led to a dramatically intensified competition. One way to circumvent cost-leadership competition is to offer individualized financial planning consulting services. In this contribution, a model to formalize financial problems and solutions under risk as an essential requirement for an IT-based financial planning process is presented, that – once implemented – may help to increase the quality of the consultation and decision support on the one hand and lower costs due to process improvements on the other hand.

1. Introduction

Due to the trend towards one-stop-shopping in the private and retail banking segments, the number of products that may be offered by a personal financial advisor as solutions to a customer's problem increased dramatically making it increasingly hard to find a superior solution. In addition, competition has intensified and customers have become more demanding. Thus, financial services providers struggle with a more difficult solution process and at the same time with shrinking margins. In recent years, many financial services providers have already found financial planning as a strategy to gain a sustainable competitive advantage at least in the customer segment of high net worth individuals.

To broaden the scope of financial planning and to offer this service to private banking and affluent customers, however, the process has to be much leaner in terms of time to come to recommendations for a specific customer. From a finance perspective the analysis and planning phase in the financial planning process, i.e. the phase where the recommendations are developed, is the most complex and demanding one. In fact, financial services providers offering financial planning services usually put a team of analysts and other experts at the task to optimize the global financial situation of a specific customer. This is a very human resources intense way of dealing with the problem, however, particularly in the domain of high net worth individuals the problems are generally of such a high complexity that IT may just support some tasks of these experts in that phase. With respect to private banking and affluent customers, the problem domain is simpler on average and often in a more structured form. This makes financial planning for these customer groups a compelling case for an appropriate system support. An underlying requirement to support this process with IT is a common language that can translate and represent the needs of the customer on the one hand (*financial problem*) and on the other hand financial products that are available to satisfy these needs (*financial solution*). In this contribution first steps to include uncertainty and risk in such a language and the solution process is proposed.

The remainder of the paper is organized as follows. In the next section related research is briefly discussed. Sec. 3 discusses the proposed problem solution process. Sec. 4 presents the basic model without risky cash flows. Sec. 5 covers starting points to include uncertainty and risk in the model. The model's applicability and limitations are discussed in Sec. 7. The main findings are briefly summarized in Sec. 8.

2. Related research

The basic idea of the presented model is based on works due to Hax in the 70s (see e.g. [11]). The main commonness between these enterprise modeling approaches and the model presented here is that both apply linear equations and matrix algebra. However, the pretension in the model presented here is a much more modest one. In the abortive enterprise modeling approaches the pretension was to model the problem completely. In this contribution it is acknowledged that the problem cannot be determined exactly in the interaction between customer and the financial consultant. Moreover, due to the complexity of the problem as well as the solution space, finding a globally *optimal* solution to a customer's problem is also not the objective here. The presented model builds on [25] and particularly extends Will's work with respect to the formalization of risk. A different approach to formalize and solve a customer's financial problems is based on graph theoretical tools (stochastic flows-with-gains approach) [9]. Monte Carlo Simulation to solve problems in the financial planning context is suggested by e.g. [18]. For a review on cash flow models see [10]. From the technical point of view, the approach presented here is particularly compatible with [5], [6], [21]. [16] deals with decision support tools in financial planning, however, their system is just able to offer "what if?" and "how to achieve?" analysis but no optimization. A financial decision support system (DSS) is also presented in [19]. The focus there is on the consideration of the investor's unique requirements and personal characteristics in the DSS and is just implemented in a prototype for the stock-selection decision.

In the following, our problem solution process is discussed as a basis for the model presented afterwards.

3. Problem Solution Process in Financial Planning

Once the data of a customer are gathered for a financial planning service, the real challenge is to come to sound recommendations with respect to the customer's situation. In the recording phase all assets and expected cash flows from salaries for instance as well as objectives and needs that will result in an alteration of the financial situation of the customer are gathered. Based on these data, interpreting the desired cash flows as restrictions, such as a constant minimal income to cover life expenditures, an optimization process is triggered. The result ideally is a transformed cash flow stream based on the cash flow restrictions of the customer that optimizes a specified objective function. From a mathematical point of view it is a linear or non-linear optimization problem subject to constraints. The objective function in combination with these constraints – both provided by the customer – are called the customer's *financial problem.*

Though the identification of the (financial) problem is a demanding task, the generation of the solution is characterized by at least the same level of complexity. On the one hand it is the task to transform vague and often qualitative needs in quantitative requirements considering cash flows, on the other hand it is the sheer uncountable number of products with often various parameters that can be included in the solution process to determine an optimal solution to the customer's problem. Talking about this solution process, apparently a global top-down optimization approach in form of an algorithm leading to a guaranteed optimal solution will hardly exist. In literature top-down approaches just exist in specific product domains. Examples are Markowitz's portfolio theory (cf. [17], *optimization through selection*) or the design of the discount in a mortgage loan (cf. [26], *optimization through configuration*). Nevertheless these optimization approaches are usually still subject to a number of restrictive assumptions.[1] In contrast to the availability of top-down domain specific optimization knowledge, top-down *combination* knowledge is rare and generally remains on a simple and abstract level.[2]

Therefore, the process to determine a good solution has to be tackled from a different and a much more modest side. If a globally optimal process is not available, it might be advantageous to combine two or more locally optimized products to form a globally superior solution. Particularly if the principle of value additivity[3] holds, locally optimized solutions can be simply summed to form a solution for the customer, which is from a mathematical point of view a very nice feature. A heuristic approach[4] that enables both the search for and the integration of *partial solutions* in a bottom-up approach as well as the utilization of available top-down combination knowledge is presented in the following. But first the term "financial solution" has to be defined in more detail.

A *financial solution* consists of a single financial product or a bundle of financial products. If a solution satisfies all constraints, it is called a *feasible solution*. Note that a feasible solution is by no means also a *superior solution* upfront. A feasible solution just satisfies all formulated constraints. In an additional step, the superior solution has to be identified applying the objective function to a set of feasible solutions that were generated during the solution process. Thus, a superior solution is defined as the best solution from a set of feasible solutions with respect to the objective function. The term "superior solution" is used intentionally instead of "optimal solution" to make clear that the superior solution hopefully will be near the (theo-

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[1] For instance Markowitz portfolio models generally assume discrete returns as normally distributed. However, there is a lot empirical evidence that this assumption is not realistic. Cf. [3] for a discussion of the impact if log returns are fat tailed.

[2] An example might be the CAPM, which includes a risk free investment opportunity (Tobin separation). As an approximation for this risk free investment opportunity often Treasury bills are considered (cf. [4]). However, there are Treasury bills with different maturities as well as different interest rates and thus with different liquidity effects for the customer. These unique characteristics of each Treasury bill are not captured in the CAPM.

[3] Cf. [4].

[4] On problem solution algorithms cf. e.g. [8], [13], on heuristic approaches cf. e.g. [12], [15] and [20]. The approach presented here belongs to the group of exact heuristic methods, which are suited for an implementation in an information system due to the fact that the problem may be poorly structured but it is well-defined; cf. [8].

retically) optimal solution however there is no guarantee that the heuristic ensures that an optimal solution is found.

If no global optimum can be easily determined top-down, at least knowledge about a local optimum within a specific product domain can be incorporated bottom-up in a (global) solution. In these cases it can be advantageous to include *partial solutions* intentionally even if they are not feasible. The *residual problem* that generally remains if such locally optimized solutions are integrated in the overall solution can be solved in another solution step. Two or more combined partial solutions may solve the (global) problem. One iteration in the process of the determination of a solution is called a *partial solution process step*.

But the proposed heuristic does not only provide for a bottom-up approach but also for the opportunity to integrate top-down combination knowledge. If such knowledge exits and a problem or partial problem is identified as one where top-down combination knowledge is present and can be applied, the system has to recognize that fact and trigger a separation of the problem into *partial problems* – if necessary.[5] This part of the solution process is called a *process of recognition* (top-down) as opposed to the *process of search*[6] for another partial solution (bottom-up).[7] In conjunction the solution process is a *hybrid process of search and recognition*[8]. This way of producing superior solutions has a number of merits: [9]

- Established local combination and optimization knowledge is incorporated into the solution process. Thus, knowledge that is already available can be utilized.
- New innovative solutions – solutions that no one would have thought of upfront – can be found due to the iterative process of search.
- Since a set of feasible solutions is generated during the solution process, the financial advisor has a number of solutions that may be presented to the customer. This has at least two advantages: First, the customer has a choice and that is generally already associated with utility. Instead, if a global top-down solution could be determined, just one solution would be offered. Second, a financial solution just considers quantitative factors, but a decision of a customer will be made based on quantitative as well as qualitative considerations. Thus, a customer might choose intentionally a second or third best solution from a quantitative point of view.

The problem solution process and the interrelations of the above described terms *partial solution*, *residual problem*, *objective function*, *superior solution* and *financial problem* are illustrated in Fig. 1.

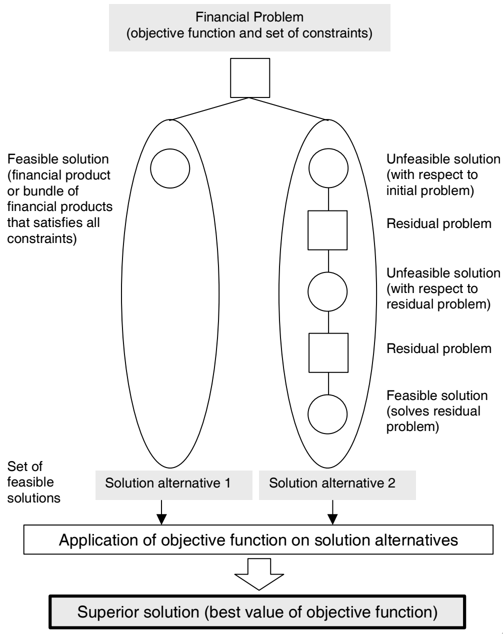

Figure 1. Schematic problem solution process[10]

A basic requirement for such a solution process being implemented is the formal representation of problems as well as solutions. As Will (cf. [25]) showed, it is advantageous to model problems as well as solutions as cash flows. Using a formal way of representing problems facilitates the use of an appropriate application that may help to find a superior solution. Therefore, an objective has to be translated into a form where the problem is characterized by a desired cash flow stream. The following simple example shall illustrate a typical customer problem.[11]

Example 1: Mr. Smith wants to undertake a longer journey in two years. Therefore, he plans to invest today and in one year 10,000 Euro each. His objective is to maximize the repayment in two years.

However, future cash flows are usually not certain but inherently affiliated with risk. This holds true on the one hand for investment products such as bonds, stocks or

[5] For instance in the ALLFIWIB project this has been realized by an autonomous so-called *combination agent*; cf. [6]. Combination knowledge will not be covered here, since the formulation and solution of customer problems that take uncertainty and risk into account are the focus at this point.

[6] This is also denoted as *learning by discovery*, see [15] and [12].

[7] Note that the process of recognition and the process of search are not separated in a way that *either* it is searched *or* available combination knowledge is applied but the solution process can be a combination of both.

[8] Cf. e.g. [6].

[9] Cf. e.g. [25].

[10] The general process pattern is taken from [21] and has been modified. In the graph the *process of recognition* is not illustrated, since it will not be the focus in this contribution.

[11] Obviously this is a very simple example in comparison to real world financial planning problems. Still, it shall suffice here in order to illustrate the model. The example will be continued throughout this contribution.

funds. On the other hand, a customer is hardly able to formulate an exact cash flow requirement in 25 years from now. However, he might be able to state at least a minimal payment that he will need. Or he might be able to set a maximum cash outflow that he is willing to bear.

Example 2: Mr. Smith not only wants to maximize the repayment in two years but he demands at least 22,000 Euro as a minimal repayment.

Another less restrictive constraint would be that a specified cash inflow has to be exceeded with a specified probability. Equally, a specified cash outflow must not be exceeded with a specified probability.

Example 3: Mr. Smith expects a repayment of more than 22,000 Euro with a probability of 90%.

Example 2 and Example 3 illustrate two different approaches of formulating uncertain constraints. In decision science Example 2 would be called a *situation under uncertainty*. There are no probabilities associated with different states of the world. The situation in Example 3 would be called a *situation under risk*. Objective or subjective probabilities can be assigned to each state of the world. Instead of using the expression "state of the world" in the following, the expression "scenario" will be used. In a meeting with a customer often "best-", "average-", and "worst-"scenarios are used to visualize uncertainty or risk in a financial planning situation.

But it is not only the customer who has desires that cannot be expressed by fixed or arbitrary cash flows but also financial products inherently contain risk. The level of future payments is – depending on the type of security or contract – generally not certain but inherently affiliated with risk. Increased return is usually combined with increased risk of an investment.[12] To configure superior solutions, it is important to also consider risky securities in the solution process, thus the model shall also be capable of taking this fact into account.

Having described the perspective on financial problems and solutions, in the following the basic model is presented.

4. Basic Model[13]

4.1 Assumptions

In the following basic assumptions and notation are introduced to lay the ground and define the restrictions for the proposed (mathematical) formulation of the solution process.

[12] Cf. e.g. [22].

[13] This section is mainly based on [25].

(AF) Framework: Future states of the world are denoted as scenarios. In each scenario $j = 1,\dots, m$ there are certain payments[14] at each point in time $t = 1,\dots, n$.[15]

(AS) Solution: Solutions are represented as (n x 1)-column vectors, where each row marks a cash inflow (positive) or a cash outflow (negative) at a specific point in time t. The solution vector $\vec{\mathbf{s}}^{ja}$ is an aggregation of $l = 1,\dots, b$ partial solutions of a solution alternative $a \in \mathbb{N}^+$ for each point in time t in a scenario j, hence an aggregation of the partial solution vectors $\vec{\mathbf{s}}^{jal}$, thus $\vec{\mathbf{s}}^{ja} = \sum_{l=1}^{b} \vec{\mathbf{s}}^{jal}$. s^{al} denotes the set of all scenario-specific partial solution vectors of partial solution l, thus $s^{al} = \{\vec{\mathbf{s}}^{1al}, \vec{\mathbf{s}}^{2al}, \dots, \vec{\mathbf{s}}^{mal}\}$. s^{a} denotes the set of all scenario-specific solution vectors of a solution alternative a, thus $s^{a} = \{\vec{\mathbf{s}}^{1a}, \vec{\mathbf{s}}^{2a}, \dots, \vec{\mathbf{s}}^{ma}\}$.

(APr) Problem: The equality and inequality constraints of the optimization problem are modeled using a (n x n) problem matrix[16] $\mathbf{P}^j$ and a (n x 1) problem vector $\vec{\mathbf{p}}^{j}$.[17] If a problem cannot be solved after a first solution step ($l = 1$) a residual problem remains denoted by the residual problem vector $\vec{\mathbf{p}}^{ja(l+1)}$ within a solution alternative s^{al} and solution step l in scenario j.

(AV) Value additivity: All cash flow streams are based on the *principle of value additivity*, i.e. "the value of the whole is equal to the sum of the values of the parts".[18] That has to be true for *within* a partial solution as well as *across* partial solutions, i.e. cash flow streams can be summed.[19]

Example 4[20]: There are three scenarios (best ($j = 1$), average ($j = 2$), and worst ($j = 3$)). An investment today of 10,000 Euro in a fund with European bonds, that is sold two years from now yields 12,000 Euro in the best, 11,000 Euro in the average, and 9,000 Euro in the worst case. This situation may be a partial solution s^{11} ($l = 1$), that can be combined with other partial solutions to form a solution alternative ($a = 1$)

[14] Within a scenario payments are assumed to be certain.

[15] In the following, pre or after tax payments will not be explicitly distinguished.

[16] The problem matrix is in case of certainty and uncertainty independent of scenarios, i.e. $\mathbf{P}^j$ will be the same for all scenarios. However, in case of risk this changes. Therefore, the problem matrix is already introduced as scenario specific at this point.

[17] See Eq. (1) and Eq. (5) to see how the coefficients form a set of linear equations that can be gathered in a problem matrix and in a problem vector.

[18] Cf. [4].

[19] Note that if the marginal tax rate is an endogenous variable, a simple aggregation of two or more after tax payment streams is not possible; cf. [25]. Therefore, in the following it is implicitly assumed that the investor's marginal tax rate is exogenously given.

[20] In this and all following examples, the three zeros for thousand are omitted in vectors and matrices for reasons of clarity and simplicity. Hence, for instance 10 means 10,000 in a vector or matrix.

$$s^{al} = s^{11} = \left\{ \vec{\mathbf{s}}^{111} = \begin{pmatrix} -10 \\ 0 \\ 12 \end{pmatrix}, \vec{\mathbf{s}}^{211} = \begin{pmatrix} -10 \\ 0 \\ 11 \end{pmatrix}, \vec{\mathbf{s}}^{311} = \begin{pmatrix} -10 \\ 0 \\ 9 \end{pmatrix} \right\}$$

4.2 The Financial Problem

As mentioned above, the financial problem consists of an objective function subject to a number of constraints. A feasible solution has to satisfy all constraints. These constraints can be represented in a system of linear equations – one equation for each point in time t:

$$P_{t1}^{j} s_1^{jal} + \ldots + P_{tt'}^{j} s_{t'}^{jal} + \ldots + P_{tt}^{j} s_t^{jal} + \ldots + P_{tn}^{j} s_n^{jal} + p_t^{j} = 0 \qquad (1)$$

If the coefficients P_{ti}^{j} and p_t^{j} are appropriately chosen, the following desired cash flow streams can be formalized:[21]

- Fixed payment (Case I): Let k denote the desired value of a payment at time t then only solutions s^{al} are feasible if and only if payment s_t^{jal} has the value $k \in$ IR across all scenarios (see Example 1). This can be represented in the following way:
  $P_{tt}^{j} = -1, P_{ti}^{j} = 0$ for $i = 1,\ldots,n; i \neq t; j = 1,\ldots,m$
  $p_t^{j} = k$ for $j = 1,\ldots,m$
  Rearranging Eq. (1) yields $s_t^{jal} = k$.
- Arbitrary payment (Case II)[22]: Feasible are all solutions s^{al} independent of the value of the payment s_t^{jal}. Consequently
  $P_{ti}^{j} = 0$ for $i = 1,\ldots,n; j = 1,\ldots,m$
  $p_t^{j} = 0$ for $j = 1,\ldots,m$
  Rearranging Eq. (1) yields $0 s_t^{jal} = 0$, which is always true.
- Desired payment is a multiple of a preceded payment (Case III): Let t' denote the preceded point in time ($t' < t$), then all solutions s^{al} are feasible if and only if s_t^{jal} has the value $\alpha \cdot s_{t'}^{jal}$, $\alpha \in$ IR, across all scenarios. Thus,
  $P_{tt}^{j} = -1, P_{tt'}^{j} = \alpha, P_{ti}^{j} = 0$ for $i = 1,\ldots,n; i \neq t; i \neq t'; j = 1,\ldots,m$
  $p_t^{j} = 0$ for $j = 1,\ldots,m$
  Rearranging Eq. (1) yields $s_t^{jal} = \alpha \cdot s_{t'}^{jal}$.

For each point in time t a constraint in form of the cases (I) – (III) can be formulated and results in n equations in the form of Eq. (1). All coefficients P_{ti}^{j} and p_t^{j} can be summarized in the problem matrix $\mathbf{P}^{j}$ and the problem vector $\vec{\mathbf{p}}^{j}$, respectively. Thus, for each of the m scenarios there is one problem matrix and one problem vector. A solution is feasible if and only if it satisfies all constraints, i.e. if Eq. (2) holds true.

$$\underbrace{\begin{pmatrix} P_{11}^{j} & \cdots & P_{1t}^{j} & \cdots & P_{1n}^{j} \\ \vdots & \ddots & \vdots & & \vdots \\ P_{t1}^{j} & \cdots & P_{tt}^{j} & \cdots & P_{tn}^{j} \\ \vdots & & \vdots & \ddots & \vdots \\ P_{n1}^{j} & \cdots & P_{nt}^{j} & \cdots & P_{nn}^{j} \end{pmatrix}}_{\text{Problem matrx}} \underbrace{\begin{pmatrix} s_1^{jal} \\ \vdots \\ s_t^{jal} \\ \vdots \\ s_n^{jal} \end{pmatrix}}_{\text{Solution vector}} + \underbrace{\begin{pmatrix} p_1^{j} \\ \vdots \\ p_t^{j} \\ \vdots \\ p_n^{j} \end{pmatrix}}_{\text{Problem vector}} = \mathbf{P}^{j} \vec{\mathbf{s}}^{jal} + \vec{\mathbf{p}}^{j} = \vec{\mathbf{0}} \qquad (2)$$

Example 5: Mr. Smith financial problem based on Example 1 can be formalized using the above notation. Taking into account that Example 1 assumed just one scenario (situation under certainty), thus $j = m = 1$, the system of equations according to Eq. (1) can be summarized in a problem matrix and problem vector (see Eq. (2))

$$\underbrace{\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\mathbf{P}^1} \vec{\mathbf{s}}^{1al} + \underbrace{\begin{pmatrix} -10 \\ -10 \\ 0 \end{pmatrix}}_{\vec{\mathbf{p}}^1} = \vec{0}.$$

4.3 Formulation and Solution of Residual Problems

As already mentioned above, it may often be advantageous to utilize local optimization knowledge to configure or select a partial solution that does not solve the initial problem entirely but yields a residual problem. Such a partial solution is called an *unfeasible solution*.

Let s^{11} denote an unfeasible solution. Apparently, a partial solution s^{12} that solves the residual problem constitutes a global solution s^{1} which solves the initial problem. The respective problem vector is determined using Eq. (3).

$$\vec{\mathbf{p}}^{ja(l+1)} := \mathbf{P}^{j} \vec{\mathbf{s}}^{jal} + \vec{\mathbf{p}}^{jal} \qquad (3)$$

Generally, the problem vector $\vec{\mathbf{p}}^{ja(l+1)}$ refers to the residual problem that remains after l partial solution process steps. To be precise, $\vec{\mathbf{p}}^{jal}$ has to be set equal to the initial problem vector for the first partial solution process step ($l = 1$), thus

$$\vec{\mathbf{p}}^{jal} := \vec{\mathbf{p}}^{j} \quad \text{for } l = 1 \qquad (4)$$

Suppose Eq. (2) yields the zero vector then the solution process is terminated. If Eq. (2) does not yield the zero vector another iteration using problem vector $\vec{\mathbf{p}}^{ja(l+1)}$ (Eq. (3)) can be performed integrating another partial solution s^{l+1}. This process can be iterated either until there is no residual problem anymore or a specified stopping rule fires, leading to a termination of this solution process without a feasible solution. A stopping rule may

[21] Constraints in the form of the following Cases I – III and later on also Cases IV and V have to be satisfied, of course, for the global solution s^{a}. However, since upfront it is not known whether the first solution process step will yield a feasible solution, s^{a} is replaced by s^{al} in the following.

[22] This case is particularly useful if investment problems have to be formulated where the future cash inflows are known but not the amount that has to be invested.

be that either a specified CPU time or a specified number of financial products (or product groups) to solve the problem is exceeded. Especially the latter rule strongly depends on the sophistication level of the customer. There the customer model briefly touched on above comes into play again. To provide tailored solutions, knowledge about the customer has to be used in the solution generation process.

After the basic model has been introduced, the center of interest will now be the inclusion of uncertainty into the model.

5. Model under Uncertainty

To formalize desired cash flows of customers that include a minimal cash inflow or a maximal cash outflow (see Example 2) another case has to be introduced that leads to inequalities in the system of linear equations. Uncertainty[23] is captured providing for $m > 1$ different scenarios.[24] Even though there is knowledge about different scenarios, there are no subjective or objective probabilities that may be assigned to each of the scenarios.

5.1 The Financial Problem

A constraint in the form of an inequality at point in time t may be formalized using m inequalities of the following type:

$$P_{t1}^j s_1^{jal} + \ldots + P_{tn}^j s_n^{jal} + p_t^j \leq 0 \tag{5}$$

Accordingly, the so-called *inequality constraint* can be described as follows.

- Desired payment is a minimum cash inflow or a maximum cash outflow (Case IV): Let v denote the desired minimum or maximum payment, then all solutions s^{al} are feasible if s_t^{jal} has at least the value v across all scenarios.[25] Thus,
  $P_{tt}^j = -1, P_{ti}^j = 0$ for $i = 1,\ldots,n; i \neq t; j = 1,\ldots,m$
  $p_t^j = v$ for $j = 1,\ldots,m$
  Rearranging Eq. (5) yields $s_t^{jal} \geq v$ for all scenarios j.

Since there may now be equalities in the form of Eq. (1) as well as inequalities in the form of Eq. (5), a $(1 \text{ x } n)$-inequality row vector $\vec{\mathbf{u}}^T$ has to be introduced to distinguish between fixed payments on the one hand (Cases I and III) and minimum, maximum or arbitrary payments on the other hand (Cases II and IV). Therefore, for each payment according to the Cases I and III u_t is set to one ($u_t = 1$). For the other two cases u_t is set to zero ($u_t = 0$).[26]

Even though the coefficients can be gathered again in the problem matrix $\mathbf{P}^j$ and the problem vector $\vec{\mathbf{p}}^j$, there are now two steps necessary to check whether all constraints according to the Cases I – IV are satisfied. In a first step it is checked whether the inequalities hold true. In a second step it is checked whether fixed payment requirements are satisfied. These two steps have to be performed for each scenario.

Step 1: To check whether the inequalities of the constraints are satisfied (Case IV), the left hand side of (6) has to be smaller or equal to the zero vector.

$$\mathbf{P}^j \vec{\mathbf{s}}^{jal} + \vec{\mathbf{p}}^{jal} \leq \vec{\mathbf{0}} \tag{6}$$

Here, all constraints are considered to be inequalities and it is checked whether at least the desired cash inflow or at most the desired cash outflow holds true for the respective solution.

Step 2: Further, using the inequality vector the fixed payment constraints (Cases I and III) are checked. Let $\mathbf{E}_{ij}$ denote the $(n \text{ x } n)$ matrix that has all elements equal to zero except for the (i,j)-th's element which is equal to one and let $\vec{\mathbf{i}}$ denote the $(n \text{ x } 1)$ vector that has elements equal to one. $\mathbf{K}$ denotes the $(n \text{ x } n)$ matrix which is yielded by a right hand sided multiplication of the left hand side of Eq. (2) with the inequality vector $\vec{\mathbf{u}}^T$.

$$\left(\mathbf{P}^j \vec{\mathbf{s}}^{jal} + \vec{\mathbf{p}}^{jal}\right)\vec{\mathbf{u}}^T = \mathbf{K} \tag{7}$$

Using Eq. (7) it can be checked whether all fixed payment constraints are satisfied.

$$\left(\sum_{t=1}^{n} \mathbf{E}_{tt}\mathbf{K}\mathbf{E}_{tt}\right)\vec{\mathbf{i}} = \vec{0} \tag{8}$$

5.2 Formulation and Solution of Residual Problems

If one of these two steps described above is not satisfied, Eq. (3) yields the residual problem. The initial problem matrix $\mathbf{P}^j$ and the inequality vector $\vec{\mathbf{u}}^T$ are not altered and can be used for the next partial solution process step.

[23] Uncertainty is defined as the absence of knowledge for the decision maker about the probability distribution on states of the world. This does not necessarily mean that these probabilities are not available at all. It just states that a decision maker has no knowledge and no subjective expectation about these probabilities. This separation is originally due to [14]. Though this separation is still widely used, it is criticized e.g. in [2].

[24] Cf. e.g. [24].

[25] This case makes also sense in the model under certainty, i.e. if there is just one scenario. The solution process cannot be performed using Eq. (2) but the two step solution process using Eq. (6) – (8) has to be applied.

[26] If there are several different desired payments at one point in time, Case IV is more binding than Cases I and III, and these for their part are more binding than Case II. Hence, Case II is overwritten by Cases I and III, and these are overwritten by Case IV. This can occur if a customer mentally distinguishes several financial problems.

6. Model under Risk[27]

Though introducing different scenarios into the consulting and solution process marks a significant improvement compared to the status quo, scenarios without scenario probabilities will not suffice for a number of financing and especially investment problems.

From the perspective of the customer inequality constraints (Case IV) may be too restrictive since a payment *must not* fall below a specified value. To make sure that this specified value is reached at all costs, the customer may have to sacrifice a lot of potential return. Especially in the context of financial planning services, the used "best" and "worst" scenarios are often very unlikely compared to the "average" scenario, since they are usually based on historical data and mark the worst and best possible outcome over a couple of years or even decades. In addition, generally speaking at least subjective probabilities for scenarios can be obtained from historical data for most traded securities. From the perspective of the solution and decision process, all relevant information that is accessible (without prohibitive costs) should be included in the process to improve the quality of the decision.

6.1 The Financial Problem

The solution process is more difficult compared to the models under certainty and uncertainty. In contrast to the constraints of Case I to IV a *probability constraint* can not be formalized using linear equations or inequalities because it does not address a specific cash flow at one point in time t but a discrete random variable characterized by all scenario specific cash flows at one point in time t and the probabilities of the scenarios. Thus, the solution process considering probability constraints could not be performed solely by matrix algebra and another assumption is necessary.

(AD) Distribution function and scenario probabilities: The payment at time t within a (global) solution s^a is a discrete probability variable denoted by S_t^a. The corresponding distribution function is denoted by $\mathrm{F}_t^a(x)$. Let w^j denote the probability of occurrence of scenario j, with $\sum_j w^j = 1;\ w^j \geq 0 \quad \forall j$. This probability is assumed to be constant in time and independent of all partial solutions s^{al} and all other solution alternatives.

[27] The model under risk below distinguishes itself just by the introduction of probabilities of occurrence for each scenario. Thus, risk is captured in a discrete function. There is no separation between systematic and unsystematic risk [4]. The focus is again to ensure minimum cash inflows or maximum cash outflows, i.e. the shortfall risk remains the center of interest. Other risk parameters such as beta, volatility, residual volatility, correlation coefficient, tracking error are at least not covered in the constraints.

To capture cases that are similar to the one described in Example 3, another two cases have to be introduced:

- Desired payment is a maximum cash outflow with a maximal probability (Case Va): If v_t denotes the desired maximum cash outflow at time t with the maximal probability w_t^v, then all solutions s^a are feasible if and only if
  $W(S_t^a \leq v_t) \leq w_t^v \Leftrightarrow \mathrm{F}_t^a(v_t) \leq w_t^v$.
  $W(S_t^a \leq v_t)$ denotes the probability that S_t^a yields a value that is equal to or below v_t. Even though probability constraints are checked without using matrix algebra, the coefficients of the problem matrix and the problem vector still have to be set to zero for further calculations, thus
  $P_{ti}^j = 0 \qquad$ for $i = 1,\ldots,n;\ j = 1,\ldots,m$
  $p_t^j = 0 \qquad$ for $j = 1,\ldots,m$
  Rearranging Eq. (1) yields $0 s_t^{jal} = 0$, which is always true.
- Desired payment is a minimum cash inflow with a minimal probability (Case Vb): If v_t denotes the desired minimum cash inflow at time t with the minimal probability w_t^{v*}, then all solutions s^a are feasible if and only if
  $W(S_t^a > v_t) \geq w_t^{v*} \Leftrightarrow \mathrm{F}_t^a(v_t) \leq \underbrace{1 - w_t^{v*}}_{w_t^v}$.
  Obviously, Case Vb can be transformed into a formulation analogously to Case Va. Analogously to Case Va, the coefficients of the problem matrix and the problem vector are set to zero.
  $P_{ti}^j = 0 \qquad$ for $i = 1,\ldots,n;\ j = 1,\ldots,m$
  $p_t^j = 0 \qquad$ for $j = 1,\ldots,m$
  Rearranging Eq. (1) yields $0 s_t^{jal} = 0$, which is always true.

To check a solution s^a on feasibility with respect to a formulated probability constraint at a time t, first the distribution function $\mathrm{F}_t^a(x)$ has to be calculated. Solution s^a comprises all partial solutions s^{al} that have been integrated in s^a so far on the way to find a feasible solution after l partial solution process steps. A separated calculation for partial solutions, like in Sec. 4.2 and Sec. 5.2 does not suffice here anymore.

Each solution alternative s_t^a at time t is characterized by its payments s_t^{ja} in the various scenarios j and the respective probabilities of occurrence w^j. Summarizing the payments and the respective probabilities into a tupel, a solution for time t (the discrete probability variable) can be written as

$$S_t^a = \left[\left(s_t^{1a}; w^1\right) \ \left(s_t^{2a}; w^2\right) \ \ldots \ \left(s_t^{ma}; w^m\right)\right] \tag{9}$$

To calculate the distribution function, first, the row of tupels has to be sorted ascending dependent on the value of the payment s_t^{ja}. The respective sorting function is denoted by Θ. After the sorting, the resulting tupels have the form $\left(s_{t,c}^a; w_{t,c}; j_{t,c}\right)$, where c denotes the rank among the tupels after the sorting took place and $j_{t,c}$ denotes the rank according to the scenarios before sorting. The coefficient t in $w_{t,c}$ reflects for which point in time the sorting took place.

$$\Theta\left[\left(s_t^{1a}; w^1\right) \ \left(s_t^{2a}; w^2\right) \ \ldots \ \left(s_t^{ma}; w^m\right)\right] = \left[\left(s_{t,1}^a; w_{t,1}; j_{t,1}\right) \ \ldots \ \left(s_{t,m}^a; w_{t,m}; j_{t,m}\right)\right] \tag{10}$$

Having sorted the tupels, now an accumulation of the probabilities is necessary to get the distribution function. This operation is denoted by Φ.

Apparently, the constraint $\mathrm{F}_t^a(v_t) \le w_t^v$ is satisfied if point $\left(v_t; w_t^v\right)$ is located *on or above* the distribution function. To check whether the probability constraints are satisfied at time t the first tupel $\left(s_t^*; w_t^*; j_t^*\right)$ has to be considered where the cumulated probability is above w_t^v. Thus, a condition of the form $\mathrm{F}_t^a(x) \le w_t^v$ is satisfied if and only if $x < s_t^*$. That is, for $\mathrm{F}_t^a(v_t) \le w_t^v$ to hold, the following statement has to be true.

$$v_t < s_t^* \Leftrightarrow s_t^* - v_t > 0 \tag{11}$$

Like in the simpler cases mentioned above, there may remain residual problems to be solved (see e.g. Example 6). How can a residual problem formally be described?

6.2 Formulation and Solution of Residual Problems

If the condition $s_t^* - v_t > 0$ (Eq. 11) is not true, this is equivalent to the statement that the solution so far provides for a payment that is too low in scenario j_t^* at time t. Therefore, for another partial solution $s^{ja(l+1)}$ at time t in scenario j_t^* the following condition – ε being some marginal value like 0.01 Euro – has to be true:

$$s_t^{j_t^* a(l+1)} > -\left(s_t^* - v_t\right) \Leftrightarrow s_t^{j_t^* a(l+1)} \ge v_t - s_t^* + \varepsilon \tag{12}$$

Apparently, Eq. (12) corresponds to Case IV and the constraints formulated there. However, in contrast to Case IV the constraint for a minimum cash inflow and a maximum cash outflow is limited to a specific scenario here. Therefore, scenario specific problem matrices $\mathbf{P}^{jal}$ have to be introduced that are dependent not only on the scenario but also on the solution alternative a and the partial solution process step l. The integration of a residual problem into the scenario specific problem matrix and problem vector is accomplished by an *adaptation matrix* $\mathbf{A}^{jal}$ and *adaptation vector* $\vec{\mathbf{a}}^{jal}$.

- For each point in time t without a probability constraint and for each point in time t with a *satisfied* probability constraint the elements of the adaptation matrix $\mathbf{A}^{jal}$ and adaptation vector $\vec{\mathbf{a}}^{jal}$ are set to zero.
  $A_{ti}^{jal} = 0 \,;\, a_t^{jal} = 0 \quad \forall i, j$
- For each point in time t with a probability constraint that is not satisfied, the elements of the adaptation matrix $\mathbf{A}^{jal}$ and adaptation vector $\vec{\mathbf{a}}^{jal}$ have to be altered according to the following rules
  $A_{ti}^{j_t^* al} = -1 ; \qquad A_{ti}^{j_t^* al} = 0 \quad \forall i \ne t; \qquad A_{ti}^{jal} = 0 \quad \forall j \ne j_t^*, i$
  $a_t^{j_t^* al} = v_t - s_t^* + \varepsilon \,; \qquad a_t^{jal} = 0 \quad \forall j \ne j_t^*$

Thus, the residual problem vector can be calculated as

$$\vec{\mathbf{p}}^{\,ja(l+1)} = \mathbf{P}^{jal} \vec{\mathbf{s}}^{\,jal} + \vec{\mathbf{p}}^{\,jal} + \vec{\mathbf{a}}^{\,jal} \tag{13}$$

and the corresponding adapted problem matrix as[28]

$$\mathbf{P}^{ja(l+1)} = \mathbf{P}^{j} + \mathbf{A}^{jal} \tag{14}$$

In contrast to Sec. 4.3 and Sec. 5.2 it is not sufficient here to check whether another partial solution just satisfies the constraints of the residual problem. Instead, it is inevitable to check the constraints also based on the complete aggregated solution, since the last integrated partial solution may alter the ranking of the tupels in Eq. (10) and thus may yield a different result based on Eq. (11).

Example 6: The probability constraint of Mr. Smith in Example 3 – to receive more than 22,000 Euro after two years ($v_3 = 22$) with a probability of at least 90% ($w_3^{v^*} = 0.9$) – corresponds to Case Vb and can formally be written as $W\left(S_3^a > 22\right) \ge 0.9 \Leftrightarrow \mathrm{F}_3^a(22) \le \underbrace{1 - 0.9}_{w_3^v} = 0.1$.

Mr. Smith is offered a funds investing in European stocks as a first (partial) solution ($l = 1$) within a solution alternative s^{21} ($a = 2$). The funds is expected to yield 35,000 Euro with 25% probability in the "best" ($w_1 = 0.25$), 25,000 Euro with 60% probability in the "average" ($w_2 = 0.6$), and 18,000 Euro with 15% probability in the "worst" scenario ($w_3 = 0.15$) in 2 years. Probability variable S_3^2 at time $t = 3$ can be written as $\left[\left(s_3^{121} = 35; w^1 = 0.25\right)\left(s_3^{221} = 25; w^2 = 0.6\right)\left(s_3^{321} = 18; w^3 = 0.15\right)\right]$. Sorting this expression yields $\Theta[(35; 0.25)(25; 0.6)(18; 0.15)] = [(18; 0.15; 3)(25; 0.6; 2)(35; 0.25; 1)]$. Cumulating these probabilities yields $\Phi[(18; 0.15; 3)(25; 0.6; 2)(35; 0.25; 1)] = [(18; 0.15; 3)(25; 0.75; 2)(35; 1; 1)]$.

This offered solution has to be checked on the probability constraint of Mr. Smith from Example 3. The relevant tupel is $\left(s_3^* = 18; w_3^* = 0.15; j_3^* = 3\right)$ and the probability constraint is $\left(v_3 = 22; w_3^v = 0.1\right)$ at time $t = 3$. The point $\left(v_3 = 22; w_3^v = 0.1\right)$, representing the probability constraint, is obviously located below the distribution function $\mathrm{F}_3^2(x)$. Thus, the probability constraint is not satisfied. Formally, Eq. (11) yields

[28] Note that in Eq. (14) it is always the initial problem matrix $\mathbf{P}^j$ that is used to determine the problem matrix for the solution step (l+1).

$s_3^* - v_3 = 18 - 22 = -4 \le 0$.

Apparently, another partial solution (l = 2) s^{22} has to provide in the "worst" scenario a cash inflow after two years (t = 3) that is greater than 4,000 Euro (v_3 = 4), i.e. $s_3^{322} > 4 \Leftrightarrow s_3^{322} \ge 4 + \varepsilon$ [29]

The constraints concerning the two fixed payments today (t = 1) and in one year (t = 2) were satisfied. To formally determine the residual problem, first the adaptation matrices $\mathbf{A}^{j21}$ and vectors $\vec{\mathbf{a}}^{j21}$ have to be determined.

$$\mathbf{A}^{121} = \mathbf{A}^{221} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \mathbf{A}^{321} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}; \vec{\mathbf{a}}^{121} = \vec{\mathbf{a}}^{221} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}; \vec{\mathbf{a}}^{331} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$$

Thus, the problem matrices $\mathbf{P}^1$ and $\mathbf{P}^2$ equal the initial problem matrix (see Example 5), whereas $\mathbf{P}^3$ is altered.

$$\mathbf{P}^1 + \mathbf{A}^{121} = \mathbf{P}^2 + \mathbf{A}^{221} = \mathbf{P}^{122} = \mathbf{P}^{222} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \mathbf{P}^j,$$

$$\mathbf{P}^{322} = \mathbf{P}^3 + \mathbf{A}^{321} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

The problem vectors in the "best" and "average" scenario for the residual problem are

$$\vec{\mathbf{p}}^{122} = \mathbf{P}^1\vec{\mathbf{s}}^{121} + \vec{\mathbf{p}}^1 + \vec{\mathbf{a}}^{121} = \vec{\mathbf{p}}^{222} = \mathbf{P}^2\vec{\mathbf{s}}^{221} + \vec{\mathbf{p}}^2 + \vec{\mathbf{a}}^{221} =$$
$$= \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} -10 \\ -10 \\ 0 \end{pmatrix} + \begin{pmatrix} -10 \\ -10 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \vec{0}.$$

Obviously, the constraints concerning the fixed payments are satisfied in these scenarios. For the problem vector in the "worst" scenario Eq. (19) yields

$$\vec{\mathbf{p}}^{322} = \mathbf{P}^3\vec{\mathbf{s}}^{321} + \vec{\mathbf{p}}^3 + \vec{\mathbf{a}}^{322} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} -10 \\ -10 \\ 0 \end{pmatrix} + \begin{pmatrix} -10 \\ -10 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}.$$

A feasible solution for the residual problem has to satisfy Eq. (6) and Eq. (8). A possible partial solution s^{22} (l = 2) for this residual problem is to sell a futures contract with a maturity of two years[30] and the following payment streams

$$s^{22} = \left\{ \vec{\mathbf{s}}^{122} = \begin{pmatrix} 0 \\ 0 \\ -8 \end{pmatrix}; \vec{\mathbf{s}}^{222} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}; \vec{\mathbf{s}}^{322} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} \right\}.$$

It can be easily shown that this partial solution satisfies Eq. (6) as well as Eq. (8) and solves the residual problem. However, this does not need to mean in turn that also a global solution has been found. The probability constraint has to be checked using the (global) solution s^2. The new probability variable S_3^2 of solution s^2 can be described as $S_3^2 = [(27;0.25)(25;0.6)(23;0.15)]$. Sorting these tupels using Eq. (10) yields $\Theta S_3^2 = [(23;0.15;3)(25;0.6;2)(27;0.25;1)]$. Now the probabilities have to be accumulated using Eq. (11): $\Phi(\Theta S_3^2) = [(23;0.15;3)(25;0.75;2)(27;1;1)]$. The relevant tupel for the check on feasibility is (23;0.15;3). Apparently, $s_3^* - v_3 = 23 - 22 = 1 \ge \varepsilon$. Thus, the global solution satisfies all constraints and solution s^2 is a feasible solution.

$$s^2 = s^{21} + s^{22} = \left\{ \vec{\mathbf{s}}^{12} = \begin{pmatrix} -10 \\ -10 \\ 27 \end{pmatrix}; \vec{\mathbf{s}}^{22} = \begin{pmatrix} -10 \\ -10 \\ 25 \end{pmatrix}; \vec{\mathbf{s}}^{32} = \begin{pmatrix} -10 \\ -10 \\ 23 \end{pmatrix} \right\}$$

So far, just the conditions to check a probability constraint have been discussed in this section. However, there may also be desired payment streams in a setting with scenarios and a probability distribution on these scenarios that correspond to the Cases I to IV. To check a solution not only on the probability but on all constraints presented above, the following conditions have to be satisfied in order to call a solution a feasible solution.

- Check equality and inequality constraints:
  - Step 1: Check inequality constraints of the (residual) problem using the last partial solution s^{al}.
  - Step 2: Check equality constraints of the (residual) problem using the last partial solution s^{al}.
- Check probability constraint: Calculate the distribution functions of solution s^a for each necessary scenario j and point in time t.

If and only if both checks are satisfied with respect to the last partial solution s^{al} and the complete solution s^a, the solution is a feasible solution s^a.

It has just been shown formally how feasible solutions can be generated in case of fixed, arbitrary, minimum and maximum payments as well as minimum payments with a minimal probability and maximum payments with a maximal probability.

7. Discussion and Limitations of the Model

The model contributes to an improvement in the quality of the consultation process in at least two ways: First, due to the obligatory starting point of the process with the financial problem of the customer, a product centric view can be circumvented. Second, the model fosters the integration of already existent local optimization knowledge. Thus, applications that have already been developed for a local optimization can still be used if the implementation provides for a sufficient modularization.

Talking about the convergence towards a superior solution, so far the model has not been implemented in the form presented above. Thus, no empirical tests could be carried out, whether a convergence can be expected. However, there are reasons for hope that the hybrid recognition and search process converges towards qualitatively good solutions. First, combination knowledge that is already available can be incorporated in the solution process. Thus, at least standard solutions that are widely offered today will be generated and in so far the model

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[29] For reasons of clarity the marginal variable is not shown in the vectors and matrices below but is only used at the end of the calculation to check whether the constraint is satisfied.

[30] Abstracting form margin payments, clearing fees, etc., there are no real cash inflows or outflows before maturity associated with the purchase of a futures contract. On futures contracts see e.g. (Steiner and Bruns 2000) or (Brealey and Myers 1996).

will *at least* ensure the status quo of the quality of recommendations in the financial services sector today. Second, if the principle of value additivity holds, it should be possible to generate feasible solutions that are favorable with respect to an evaluation function. If the net present value (NPV) is applied as objective function, suppose two locally optimized partial solutions have been generated that together do not satisfy the constraints but generate a substantially positive NPV. The third partial solution just aims at solving the residual problem. Even if this third partial solution generates a slightly negative NPV, the integrated global solution will most likely still provide for a positive NPV. Third, in the ALLFIWIB project already mentioned above, it could be shown in a prototypical implementation that superior solutions are generated and can be expected using this approach. Though in the ALLFIWIB project just the case of certainty has been covered, since the algebra is not significantly more complicated the empirical evidence might be a cautious proxy for the convergence towards a superior solution of the model under uncertainty or risk. Nevertheless, this issue is certainly still an open research question.

Besides the question of convergence, there are another two broad issues that limit the above model to some extent. First, the representation of risk can be criticized. Especially the constraints that can be formulated by the customer concerning minimum cash inflows or maximum cash outflows – eventually with a specific probability – just capture shortfall risks but do not take into account any chances. Applying an appropriate decision rule, this situation can be relaxed. If the decision rule takes into account also chances as opposed to just focusing on the downside risk, a well balanced decision can be safeguarded. Talking about the shortfall risk, it can be faulted that it is not combined with the actual shortfall loss, once the shortfall situation occurs.[31] This is a major deficiency but could be integrated relatively easy into the model. In addition, the probabilities of occurrence were assumed to be constant in time, across all scenarios and across all solutions. This may be in some instances an oversimplification, however, the introduction of time-specific probabilities into the model would not pose a big difficulty. Knowledge about correlation of two or more financial products that may be used in an optimization process can only be implicitly used between two payment streams. Thus, the opportunity of risk diversification can hardly be formalized *between* different partial solutions. Nevertheless, correlation can be accounted for explicitly *within* a partial solution. In a setting where partial solutions are calculated and proposed by independent software agents that represent a specific product domain, e.g. stocks, this does not pose a prohibitive setback for the model. Important diversification effects can be captured by this way. But, to broaden the scope to the global financial situation of the customer is still an open research issue that should be focused on in future research efforts. The issue of risk diversification is the connection to the second major limitation of the model.

Value additivity is the basic underlying of this model, which builds on the simple additivity of partial solutions, i.e. their payment streams, to form a global solution for a customer problem. The total value of a solution is the sum of its parts, i.e. its partial solutions. Along the same lines, a problem can be decoupled in partial problem components if necessary. Brealey and Myers (cf. [4]) nicely point out and show that given a perfect capital market, a firm's total value is just the sum of its parts. Thus, the value additivity principle holds: Investors are not willing to pay extra for diversification effects since they can diversify in their own portfolio. But the model was developed to work on the level of the customer, hence, where diversification has to be performed and should be an important issue. As already mentioned above, locally, i.e. concerning partial solutions, diversification can be conveniently taken into account, globally not yet. Analogously the constant marginal tax rate may in a number of cases constitute an oversimplification. It is well imaginable that a partial solution generates such high tax deductible amounts that the marginal tax rate would be lowered after the integration of this partial solution. However, this would most likely have effects on all partial solutions already integrated and also on the efficiency of the initial portfolio.

8. Conclusion

A model has been presented that allows for the inclusion of uncertainty and risk into the formulation of financial problems by the customer as well as in the solution process, i.e. intelligently bundling financial products to form a superior solution for a specific customer problem. The presented formal model is just a first step to better incorporate risk in the financial planning process and facilitate the use of IT for the solution generation process. Especially customer groups with comparably structured problems and a limited problem domain such as the Affluent segment may benefit substantially by an IT enabled financial planning concerning the solution generation process. Today, they cannot be serviced appropriately due to the prohibitive high costs, but tomorrow supported by adequate applications in combination with well-trained staff this may become a sustainable competitive advantage. Future research conducted at the Competence Center IT & Financial Services at the University of Augsburg (http://www.wi-if.de) will focus on these issues.

[31] A detailed representation and criticism of the shortfall risk can be found in [1].

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