University of Augsburg
Prof. Dr. Hans Ulrich Buhl
Research Center
Finance & Information Management
Department of Information Systems
Engineering & Financial Management

Universität Augsburg University

Discussion Paper WI-166

Decision Support for Financial Planning

by

Jochen Dzienziol, Dennis Kundisch

December 2005

in: Journal of Decision Systems, 17, 2, 2008, p. 175-209

Decision Support for Financial Planning

Jochen Dzienziol, Dennis Kundisch

University of Augsburg
Department of Information Systems & Financial Engineering
Competence Center IT & Financial Services
Universitaetsstrasse 16
86135 Augsburg
Germany
Phone: ++49-821-598-4139
Fax: ++49-821-598-4225
Email: {jochen.dzienziol|dennis.kundisch}@wiwi.uni-augsburg.de
URL: http://www.wi-if.de

Decision Support for Financial Planning

Abstract:

Financial consulting is a demanding task. Due to the complexity and fuzziness of customers' financial problems on the one hand and the amount of possible products that may be considered to configure solutions to these problems on the other hand, an adequate DSS is essential. A model is presented that allows for the inclusion of uncertainty and risk into the formulation of financial problems by the customer as well as in the solution process, i.e. intelligently bundling financial products to form a superior solution for a specific customer problem. As an innovation we introduce the transformation of probability constraints into scenario specific minimum payment constraints, which seems applicable far beyond the domain of financial planning.

Keywords:

Financial planning, financial consulting, decision support system, risk, uncertainty, scenarios

1. Introduction

The number of products that may be offered by a personal financial advisor as solution to a customer's problem increased dramatically[1] making it harder to find a superior solution. In addition, competition has intensified and customers have become more demanding [5], [7]. Thus, financial services providers struggle with a more difficult solution process and at the same time with shrinking margins. In recent years, many financial services providers have found financial planning as a strategy to gain a sustainable competitive advantage at least in the customer segment of high net worth individuals.

To broaden the scope of financial planning and to offer this service to private banking and affluent customers, however, the process has to be much leaner in terms of time to come to recommendations for a specific customer. From a finance perspective the analysis and planning phase in the financial planning process, i.e. the phase where the recommendations are developed, is the most complex and demanding one. In fact, financial services providers offering financial planning services for high net worth individuals usually put a team of analysts and other experts at the task to optimize the global financial situation of a specific customer. This is a very human resources intense way of dealing with the problem, however, particularly in the domain of high net worth individuals the problems are generally of such complexity that the use of information technology may just support some tasks of these experts in that phase. With respect to private banking and affluent customers, the problem domain is simpler on average and often in a more structured form. This makes financial planning for these customer segments a compelling case for an appropriate decision system support. An underlying requirement to support this process with a decision support system (DSS) is a common language that can translate and represent the needs of the customer on the one hand (*financial problem*) and on the other hand financial products that are available to satisfy these needs (*financial solution*). In this contribution such a language and a suitable solution process including the possibility to include risk is proposed. Moreover, the proposed model allows solving modular problems, such as pension planning or mortgage lending, much

[1] E.g. in the German retail market for financial services, as of 2005 there are far more than 6.000 open end funds as well as far more than 50.000 retail derivatives available that may potentially be a part of a solution to a financial problem.

easier compared to the status quo of models and applications in the financial services market while taking into account the whole financial situation of a customer.

The remainder of the paper is organized as follows. Section 2 reports related research in the domain of DSS research in financial planning. Section 3 presents the proposed problem solution process. Section 4 presents the basic model. In Section 5 an extension of the basic model including uncertainty and risk is proposed. The model's applicability and limitations are discussed in Section 6. Section 7 summarizes the findings.

2. Related research

In literature a lot has been written about personal financial planning[2] and about decision support [9] systems. Moreover, there are a number of contributions that deal with expert or decision support systems in corporate financial planning or in banks (e.g. [8], [13], [15], [24], [31]). However, concerning personal finance and its decision support, there is much less coverage. Locarek and Preuss present a prototypical decision support tool in financial planning, however, their system is just able to offer "what if?" and "how to achieve?" analysis, but no optimization [20]. Palm-dos-Reis and Zahedi present a DSS for private investors [25]. The focus in their contribution lies on the appropriate selection of a model for investment decisions based on a customer's preferences. Gaul proposes an approach to formalize and solve a customer's financial problem based on *graph theoretical tools* (stochastic flows-with-gains approach) [11]. *Monte Carlo Simulation* to solve problems in the financial planning context is suggested by McCabe and Boinske [23]. Another related contribution is due to Gardin et al. [10]. They propose a liquidity management approach including risk using the *simple recourse method*. Benaroch and Dhar propose a DSS using *qualitative reasoning techniques* to support the implementation of hedging strategies for professional traders [2].

[2] Whole journals have been dedicated to this task such as the Journal of Financial Planning, Journal of Accountancy, CPA Journal, Journal of Financial Service Professionals. However, these are mostly journals with a more practical "hands-on" view and without a double-blind-refereed process.

All these different approaches have their merits and deal with the solution of some kind of financial problem, but with respect to the following requisites, neither of them can fully convince: We expect our approach to be as traceable as possible; we want to be able to use the approach for financing, for investment as well as for mixed financial problems. We want to be able to benefit from already existing domain-specific knowledge and at the same time, we want to be able to find or configure financial solutions that are innovative and new. Therefore, we propose a different approach.

The basic idea of the presented model in this contribution is based on works about enterprise modeling due to Hax in the 70s (see e.g. [12]). The main commonness between these approaches and the model presented here is that both apply linear equations and matrix algebra. However, the pretension in the model presented here is a much more modest one. In the abortive enterprise modeling approaches the pretension was to model the problem completely. In this contribution it is acknowledged that the problem cannot be determined exactly in the interaction between customer and the financial consultant. Moreover, due to the complexity of the problem as well as the solution space [25], finding a globally *optimal* solution to a customer's problem is also not the objective here. The presented model extends contributions by [18], [30] with respect to the formalization and inclusion of risk. Hence, a *model-driven DSS* [26] is proposed.

From the technical point of view, the approach presented here is particularly compatible with a *blackboard approach* proposed by Hayes-Roth [14] and applied in the financial consulting context e.g. by Buhl et al. [4], Sandbiller et al. [27] and Einsfeld et al. [7].

In the following, our problem solution process is discussed as a basis for the model presented afterwards.

3. Problem Solution Process in Financial Planning

Once the data of a customer are gathered for a financial planning service, the real challenge is to come to sound recommendations with respect to the customer's situation. In the recording phase all assets and expected cash flows (salaries, dividends, consumption payouts, etc.) as well as objectives and needs that will result in an alteration of the financial situation of the customer are gathered. Based on these data, interpreting the desired cash flows as restrictions, such as a constant minimum income to cover life expenditures, an optimization process is triggered. The result ideally is a transformed cash flow stream based on the cash flow restrictions of the customer that optimizes a specified objective function. From a mathematical point of view it is a linear or non-linear optimization problem subject to constraints. The objective function in combination with these constraints – both provided by and discussed with the customer – are called the customer's *financial problem*.

Though the identification of the (financial) problem is a demanding task, the generation of the solution is characterized by at least the same level of complexity. On the one hand it is the task to transform vague and often qualitative needs in quantitative requirements considering cash flows; on the other hand it is the sheer uncountable number of products with often various parameters that can be included in the solution process to determine an optimal solution to the customer's problem [25]. Talking about this solution process, apparently a global top-down optimization approach in form of an algorithm leading to a guaranteed optimal solution will hardly exist. In literature top-down approaches just exist in specific product domains. Examples are Markowitz's portfolio theory (cf. [22], *optimization through selection*) or the design of the discount in a mortgage loan (cf. [32], *optimization through configuration*). Nevertheless these optimization approaches are usually still subject to a number of restrictive assumptions. In contrast to the availability of top-down domain specific optimization knowledge, top-down *combination* knowledge is rare and generally remains on a simple and abstract level.[3]

Therefore, the process to determine a good solution has to be tackled from a different and a much more modest side. If a globally optimal process is not available, it might be advantageous to combine two or more *locally optimized* products - or bundles of products - to form a globally superior solution.

[3] An example might be the CAPM, which includes a risk free investment opportunity (Tobin separation). As an approximation for this risk free investment opportunity often Treasury bills are considered (cf. [3]). However, there are Treasury bills with different maturities as well as different interest rates and thus with different liquidity effects for the customer. These unique characteristics of each Treasury bill are not captured in the CAPM.

Particularly if the *principle of value additivity* [3] holds, locally optimized solutions can be simply summed to form a solution for the customer, which is from a mathematical point of view a very nice feature. A heuristic approach[4] that enables both the search (on heuristic search in general see e.g. contributions in [33]) for and the integration of *partial solutions* in a bottom-up approach as well as the utilization of available top-down combination knowledge is presented in the following. But first the term "financial solution" has to be defined in more detail.

A *financial solution* consists of a single financial product or a bundle of financial products. If a solution satisfies all constraints, it is called a *feasible solution*. In an additional step, the *superior solution* has to be identified applying the objective function to the set of feasible solutions that were generated during the solution process. Thus, a superior solution is defined as the optimal solution with respect to the (incomplete) feasible set and the objective function.

If no global optimum can be easily determined top-down, at least knowledge about a local optimum within a specific product domain can be incorporated bottom-up in a (global) solution. In these cases it can be advantageous to include *partial solutions* intentionally even if they are not feasible. The *residual problem* that generally remains if such locally optimized solutions are integrated in the overall solution can be solved in another solution step. Two or more combined partial solutions may solve the (global) problem. One iteration in the process of the determination of a solution is called a *partial solution process step*.

But the proposed heuristic does not only provide for a bottom-up approach, but also for the opportunity to integrate top-down combination knowledge. If such knowledge exits and a problem or partial problem is identified as one where top-down combination knowledge is present and can be applied, the system has to recognize that fact and trigger a separation of the problem into *partial problems*

[4] On heuristic optimization in portfolio management see e.g. [21]. On problem solution algorithms cf. e.g. [6], on heuristic approaches cf. e.g. [19], [21], and [33]. The approach presented here belongs to the group of exact heuristic methods, which are suited for an implementation in an information system due to the fact that the problem may be poorly structured but it is well-defined; cf. [6].

– if necessary.[5] This part of the solution process is called a *process of recognition* (top-down) as opposed to the *process of search* (or *learning by discovery*, see [19]) for another partial solution (bottom-up). (Note that the process of recognition and the process of search are not separated in a way that *either* it is searched *or* available combination knowledge is applied but the solution process can be a combination of both.) In conjunction the solution process is a *hybrid process of search and recognition*. This way of producing superior solutions has a number of merits [17], [30]:

- Established local combination and optimization knowledge is incorporated into the solution process. Thus, knowledge that is already available and tested can be utilized.
- New innovative solutions – solutions that no one would have thought of upfront – can be found due to the iterative process of search.
- Since a set of feasible solutions is generated during the solution process, the financial advisor has a number of solutions that may be presented to the customer. This has at least two advantages: First, the customer has a choice and that is generally already associated with utility. Instead, if a global top-down solution could be determined, just one solution would be offered. Second, a financial solution just considers quantitative factors, but a decision of a customer will be made based on quantitative as well as qualitative considerations. Thus, a customer might choose intentionally a second or third best solution from a quantitative point of view.

To cope with the problem of complexity, a concept of *cooperating knowledge based systems* is used. For each financial domain a knowledge based system works as an expert (a so-called “domain-agent”) selecting and/or configuring solutions to a given (residual) problem. Implementing domain-agents as separately running software processes the performance can be improved on the one hand and the maintenance and extension of the knowledge base is simplified. For implementing this concept of

[5] For instance in the ALLFIWIB project this has been realized by an autonomous so-called *combination agent*; cf. [7]. Combination knowledge will not be covered here, since the formulation and solution of customer problems that take uncertainty and risk into account are the focus at this point.

offering combined solutions of several locally optimized products cooperation of the domain-agents is necessary [4]. The blackboard approach [14] can be applied to realize this cooperation. Each domain-agent can offer and write solutions on the "blackboard" to (residual) problems which it has taken from the blackboard upfront. This implicit way of cooperation is complemented by a "combination-agent", with explicit knowledge overlapping several domains. A control system takes care of the solution process terminating at a specific point in time, first with a request to the domain-agents to solve existing residual problems without aiming at reaching local optimality und second by breaking off the solution process after a specific time frame.

The problem solution process and the interrelations of the above described terms *partial solution*, *residual problem*, *objective function*, *superior solution* and *financial problem* are illustrated in Fig. 1.

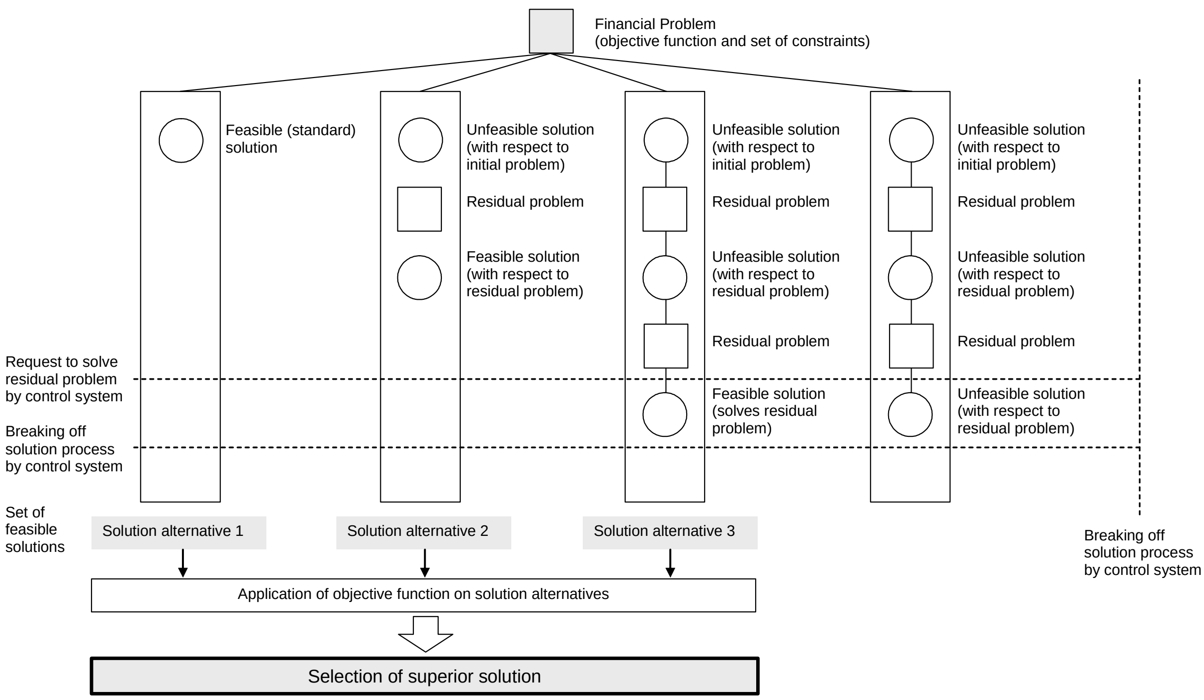


Figure 1. Schematic problem solution process[6]

A basic requirement for such a solution process being implemented is the formal representation of problems as well as solutions. As Will showed, it is advantageous to model problems as well as solutions

[6] The general process pattern is taken from [27] and has been modified and extended. For the sake of simplification, in the graph the *process of recognition* - a *combination agent* splitting a problem into two or more disjunct problems - is not illustrated, since it will not be the focus in this contribution.

as cash flows [30]. Using a formal way of representing problems facilitates the use of an appropriate DSS that may help to find a superior solution. Therefore, an objective has to be translated into a form where the problem is characterized by a desired cash flow stream. The following simple example shall illustrate a typical customer problem.[7]

Example 1*: Mr. Smith wants to undertake a longer journey in two years. Therefore, he plans to invest today and in one year 10,000 Euro each. His objective is to maximize the repayment in two years.*

However, future cash flows are usually not certain but inherently affiliated with risk. This holds true on the one hand for investment products such as bonds, stocks or funds. On the other hand, a customer is hardly able to formulate an exact cash flow requirement in 25 years from now. However, he might be able to state at least a minimal payment that he will need. Or he might be able to set a maximum cash outflow that he is willing to bear.

Example 2*: Mr. Smith not only wants to maximize the repayment in two years but he demands at least 22,000 Euro as a minimal repayment.*

Another less restrictive constraint would be that a specified cash inflow has to be exceeded with a specified probability. Equally, a specified cash outflow must not be exceeded with a specified probability.

Example 3*: Mr. Smith expects a repayment of more than 22,000 Euro with a probability of 90%.*

Example 2 and Example 3 illustrate two different approaches of formulating uncertain constraints. In decision science Example 2 refers to a *situation under uncertainty*. There are no probabilities associated with different states of the world. The constraint in Example 3 only makes sense in a *situation under risk* where objective or subjective probabilities can be assigned to each state of the

[7] Obviously this is a very simple example in comparison to real world financial planning problems; however, it is not unusual that customers come with modular and specific problems to their financial services provider (pension planning, mortgage lending, consumer finance etc.). A solution to such problems should still take into account the whole financial situation of this customer. The example will be continued throughout this contribution.

world. Instead of using the expression “state of the world” in the following, the expression “scenario” will be used. In a meeting with a customer often “best-”, “average-”, and “worst-”scenarios are used to visualize uncertainty or risk in a financial planning situation.

But it is not only the customer who has desires that cannot be expressed by fixed or arbitrary cash flows but also financial products inherently contain risk with respect to the level future payments in different scenarios. Increased return is usually combined with increased risk of an investment [28]. To configure superior solutions, it is important to also consider risky securities in the solution process, thus the model shall also be capable of taking this fact into account.

Having described the perspective on financial problems and solutions, in the following the basic model is presented.

4. Basic Model

4.1 Assumptions

In the following basic assumptions and notation are introduced to lay the ground and define the restrictions for the proposed (mathematical) formulation of the solution process [30].

(AF) Framework: Future states of the world are denoted as scenarios. In each scenario $j = 1,..., m$ there are certain payments at each point in time $t = 1,..., n$. In the following, pre or after tax payments will not be explicitly distinguished.

(AS) Solution: Solutions are represented as (n x 1)-column vectors, where each row marks a cash inflow (positive) or a cash outflow (negative) at a specific point in time t. The solution vector $\vec{\mathbf{s}}^{ja}$ is an aggregation of $l = 1,\ldots, b$ partial solutions of a solution alternative $a \in$ IN+ for each point in time t in a scenario j, hence an aggregation of the partial solution vectors $\vec{\mathbf{s}}^{jal}$, thus $\vec{\mathbf{s}}^{ja} = \sum_{l=1}^{b} \vec{\mathbf{s}}^{jal}$. s^{al} denotes the set of all scenario-specific partial solution vectors of partial solution l, thus $s^{al} = \{\vec{\mathbf{s}}^{1al}, \vec{\mathbf{s}}^{2al}, \ldots, \vec{\mathbf{s}}^{mal}\}$. s^a denotes the set of all scenario-specific solution vectors of a solution alternative a, thus $s^a = \{\vec{\mathbf{s}}^{1a}, \vec{\mathbf{s}}^{2a}, \ldots, \vec{\mathbf{s}}^{ma}\}$.

(APr) Problem: The equality and inequality constraints of the optimization problem are modeled using a (n x n) problem matrix[8] $\mathbf{P}^j$ and a (n x 1) problem vector $\vec{\mathbf{p}}^{j}$. If a problem cannot be solved after a first solution step ($l = 1$) a residual problem remains denoted by the residual problem vector $\vec{\mathbf{p}}^{ja(l+1)}$ within a solution alternative s^{al} and solution step l in scenario j.

[8] The problem matrix is in case of certainty and uncertainty independent of scenarios, i.e. $\mathbf{P}^j$ will be the same for all scenarios. However, in case of risk this changes. Therefore, the problem matrix is already introduced as scenario specific at this point.

(AV) Value additivity: All cash flow streams are based on the *principle of value additivity*, i.e. “the value of the whole is equal to the sum of the values of the parts”.[3] That has to be true for *within* a partial solution as well as *across* partial solutions, i.e. cash flow streams can be summed.[9]

***Example 4*[10]**: *There are three scenarios (best (j = 1), average (j = 2), and worst (j = 3)). An investment today of 10,000 Euro in a fund with European bonds, that is sold two years from now yields 12,000 Euro in the best, 11,000 Euro in the average, and 9,000 Euro in the worst case. This situation may be a partial solution* s^{11} *(l = 1), that can be combined with other partial solutions to form a solution alternative (a = 1)*

$$s^{al} = s^{11} = \left\{ \vec{\mathbf{s}}^{111} = \begin{pmatrix} -10 \\ 0 \\ 12 \end{pmatrix}, \vec{\mathbf{s}}^{211} = \begin{pmatrix} -10 \\ 0 \\ 11 \end{pmatrix}, \vec{\mathbf{s}}^{311} = \begin{pmatrix} -10 \\ 0 \\ 9 \end{pmatrix} \right\}$$

4.2 The Financial Problem

As mentioned above, the financial problem consists of an objective function subject to a number of constraints. A feasible solution has to satisfy all constraints. These constraints can be represented in a system of linear equations – one equation for each point in time *t*:

$$P_{t1}^{j} s_{1}^{jal} + \ldots + P_{tt'}^{j} s_{t'}^{jal} + \ldots + P_{tt}^{j} s_{t}^{jal} + \ldots + P_{tn}^{j} s_{n}^{jal} + p_{t}^{j} = 0 \tag{1}$$

If the coefficients P_{ti}^{j} and p_{t}^{j} are appropriately chosen, the following desired cash flow streams (constraints) can be formalized:[11]

- *Fixed payment* (Case I): Let *k* denote the desired value of a payment at time t then only solutions s^{al} are feasible if and only if payment s_{t}^{jal} has the value $k \in$ IR across all scenarios (see Example 1). This can be represented in the following way:

---

[9] Note that if the marginal tax rate is an endogenous variable, a simple aggregation of two or more after tax payment streams is not possible [30]. Therefore, in the following it is implicitly assumed that the investor’s marginal tax rate is exogenously given.

[10] In all examples the three zeros for thousand are omitted in vectors and matrices for reasons of clarity and simplicity.

[11] Constraints in the form of the following Cases I – III and later on also Cases IV and V have to be satisfied, of course, for the global solution s^{a}. However, since upfront it is not known whether the first solution process step will yield a feasible solution, s^{a} is replaced by s^{al} in the following.

$P_{tt}^{j} = -1, P_{ti}^{j} = 0$ for $i = 1, \ldots, n; i \neq t; j = 1, \ldots, m$
$p_{t}^{j} = k$ for $j = 1, \ldots, m$

Rearranging Eq. (1) yields $s_{t}^{jal} = k$.

- *Arbitrary payment* (Case II): Feasible are all solutions s^{al} independent of the value of the payment s_{t}^{jal}.

  Consequently

  $P_{ti}^{j} = 0$ for $i = 1, \ldots, n; j = 1, \ldots, m$
  $p_{t}^{j} = 0$ for $j = 1, \ldots, m$

  Rearranging Eq. (1) yields $0 s_{t}^{jal} = 0$, which is always true. Note that this case is particularly useful if investment problems have to be formulated where the desired future cash inflows are known but not the amount that has to be invested.

- *Desired payment is a multiple of a preceded payment* (Case III): Let *t'* denote the preceded point in time (*t' < t*), then all solutions s^{al} are feasible if and only if s_{t}^{jal} has the value $\alpha \cdot s_{t'}^{jal}$, $\alpha \in$ IR, across all scenarios. Thus,

  $P_{tt}^{j} = -1, P_{tt'}^{j} = \alpha, P_{ti}^{j} = 0$ for $i = 1, \ldots, n; i \neq t; i \neq t'; j = 1, \ldots, m$
  $p_{t}^{j} = 0$ for $j = 1, \ldots, m$

  Rearranging Eq. (1) yields $s_{t}^{jal} = \alpha \cdot s_{t'}^{jal}$.

For each point in time *t* a constraint in form of the cases (I) – (III) can be formulated and results in *n* equations in the form of Eq. (1). All coefficients P_{ti}^{j} and p_{t}^{j} can be summarized in the problem matrix **P**j and the problem vector $\vec{\mathbf{p}}^{j}$, respectively. Thus, for each of the *m* scenarios there is one problem matrix and one problem vector. A solution is feasible if and only if it satisfies all constraints, i.e. if Eq. (2) holds true.

$$\underbrace{\begin{pmatrix} P_{11}^{j} & \cdots & P_{1t}^{j} & \cdots & P_{1n}^{j} \\ \vdots & \ddots & \vdots & & \vdots \\ P_{t1}^{j} & \cdots & P_{tt}^{j} & \cdots & P_{tn}^{j} \\ \vdots & & \vdots & \ddots & \vdots \\ P_{n1}^{j} & \cdots & P_{nt}^{j} & \cdots & P_{nn}^{j} \end{pmatrix}}_{\text{Problem matrx}} \underbrace{\begin{pmatrix} s_1^{jal} \\ \vdots \\ s_t^{jal} \\ \vdots \\ s_n^{jal} \end{pmatrix}}_{\substack{\text{Solution} \\ \text{vector}}} + \underbrace{\begin{pmatrix} p_1^{j} \\ \vdots \\ p_t^{j} \\ \vdots \\ p_n^{j} \end{pmatrix}}_{\substack{\text{Problem} \\ \text{vector}}} = \mathbf{P}^{j}\vec{\mathbf{s}}^{\,jal} + \vec{\mathbf{p}}^{\,j} = \vec{\mathbf{0}} \tag{2}$$

***Example 5**: Mr. Smith financial problem based on Example 1 can be formalized using the above notation. Taking into account that Example 1 assumed just one scenario (situation under certainty), thus j = m = 1, the system of equations according to Eq. (1) can be summarized in a problem matrix and problem vector (see Eq. (2))*

$$\underbrace{\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{\mathbf{P}^1} \vec{\mathbf{s}}^{1al} + \underbrace{\begin{pmatrix} -10 \\ -10 \\ 0 \end{pmatrix}}_{\vec{\mathbf{p}}^1} = \vec{0}\,.$$

4.3 Formulation and Solution of Residual Problems

As already mentioned above, it may often be advantageous to utilize local optimization knowledge to configure or select a partial solution that does not solve the initial problem entirely but yields a residual problem. Such a partial solution is called an *unfeasible solution*.

Let s^{11} denote an unfeasible solution. Apparently, a partial solution s^{12} that solves the residual problem constitutes a global solution s^1 which solves the initial problem. The respective problem vector is determined using Eq. (3).

$$\vec{\mathbf{p}}^{\,ja(l+1)} := \mathbf{P}^{j}\vec{\mathbf{s}}^{\,jal} + \vec{\mathbf{p}}^{\,jal} \tag{3}$$

Generally, the problem vector $\vec{\mathbf{p}}^{\,ja(l+1)}$ refers to the residual problem that remains after l partial solution process steps. To be precise, $\vec{\mathbf{p}}^{\,jal}$ has to be set equal to the initial problem vector for the first partial solution process step (l = 1), thus

$$\vec{\mathbf{p}}^{\,jal} := \vec{\mathbf{p}}^{\,j} \quad \text{for } l = 1 \tag{4}$$

Suppose Eq. (2) yields the zero vector then the solution process is terminated. If Eq. (2) does not yield the zero vector another iteration using problem vector $\vec{\mathbf{p}}^{\,ja(l+1)}$ (Eq. (3)) can be performed integrating another partial solution $s^l + 1$. This process can be iterated either until there is no residual problem anymore or a specified stopping rule fires, leading to a termination of this solution process without a feasible solution. A stopping rule may be that either a specified CPU time or a specified number of financial products (or product groups) to solve the problem is exceeded. Especially the latter rule strongly depends on the sophistication level of the customer. There the customer model briefly touched on above comes into play again. To provide tailored solutions, knowledge about the customer has to be used in the solution generation process.

After the basic model has been introduced, the center of interest will now be the inclusion of uncertainty into the model.

5. Extensions: Model under Uncertainty and Risk

In the following sections the basic model (Sec. 4) is extended first to capture uncertainty (Sec. 5.1) and finally to capture risk (Sec. 5.2).

5.1 Model under Uncertainty

To formalize desired cash flows of customers that include a minimal cash inflow or a maximal cash outflow (see Example 2) another case has to be introduced that leads to inequalities in the system of linear equations. Uncertainty is captured providing for $m > 1$ different scenarios [29]. Even though there is knowledge about different scenarios, there are no subjective or objective probabilities that may be assigned to each of the scenarios. Uncertainty is defined as the absence of knowledge for the decision maker about the probability distribution on states of the world. This does not necessarily mean that these probabilities are not available at all. It just states that a decision maker has no knowledge and no subjective expectation about these probabilities. (This separation is originally due to [16]. Though this separation is still widely used, it is criticized e.g. in [1].)

5.1.1 The Financial Problem

A constraint in the form of an inequality at point in time t may be formalized using m inequalities of the following type:

$$P_{t1}^{j} s_1^{jal} + \ldots + P_{tn}^{j} s_n^{jal} + p_t^{j} \leq 0 \tag{5}$$

Accordingly, the so-called *inequality constraint* can be described as follows.

- *Desired payment is a minimum cash inflow or a maximum cash outflow* (Case IV): Let v denote the desired minimum or maximum payment, then all solutions s^{al} are feasible if s_t^{jal} has at least the value v across all scenarios.[12] Thus,

  $$P_{tt}^{j} = -1, P_{ti}^{j} = 0 \qquad \text{for } i = 1, \ldots, n; i \neq t; \; j = 1, \ldots, m$$
  $$p_t^{j} = v \qquad \text{for } j = 1, \ldots, m$$

  Rearranging Eq. (5) yields $s_t^{jal} \geq v$ for all scenarios j.

Since there may now be equalities in the form of Eq. (1) as well as inequalities in the form of Eq. (5), a (1 x n)-inequality row vector $\vec{\mathbf{u}}^T$ has to be introduced to distinguish between fixed payments on the one hand (Cases I and III) and minimum, maximum or arbitrary payments on the other hand (Cases II and IV). Therefore, for each payment according to the Cases I and III u_t is set to one (u_t = 1). For the other two cases u_t is set to zero (u_t = 0). If there are several different desired payments at one point in time, Case IV is more binding than Cases I and III, and these for their part are more binding than Case II. Hence, Case II is overwritten by Cases I and III, and these are overwritten by Case IV. This can occur if a customer mentally distinguishes several financial problems.

Even though the coefficients can be gathered again in the problem matrix $\mathbf{P}^j$ and the problem vector $\vec{\mathbf{p}}^{j}$, there are now two steps necessary to check whether all constraints according to the Cases I – IV are satisfied. In a first step it is checked whether the inequalities hold true. In a second step it is

---

[12] This case makes also sense in the model under certainty, i.e. if there is just one scenario. The solution process cannot be performed using Eq. (2) but the two step solution process using Eq. (6) – (8) has to be applied.

checked whether fixed payment requirements are satisfied. These two steps have to be performed for each scenario.

Step 1: To check whether the inequalities of the constraints are satisfied (Case IV), the left hand side of (6) has to be smaller or equal to the zero vector.

$$\mathbf{P}^{j}\vec{\mathbf{s}}^{\,jal}+\vec{\mathbf{p}}^{\,jal}\leq\vec{\mathbf{0}} \tag{6}$$

Here, all constraints are considered to be inequalities and it is checked whether at least the desired cash inflow or at most the desired cash outflow holds true for the respective solution.

Step 2: Further, using the inequality vector the fixed payment constraints (Cases I and III) are checked. Let $\mathbf{E}_{ij}$ denote the (n x n) matrix that has all elements equal to zero except for the (i,j)-th's element which is equal to one and let $\vec{\mathbf{i}}$ denote the (n x 1) vector that has elements equal to one. **K** denotes the (n x n) matrix which is yielded by a right hand sided multiplication of the left hand side of Eq. (2) with the inequality vector $\vec{\mathbf{u}}^{T}$.

$$\left(\mathbf{P}^{j}\vec{\mathbf{s}}^{\,jal}+\vec{\mathbf{p}}^{\,jal}\right)\vec{\mathbf{u}}^{T}=\mathbf{K} \tag{7}$$

Using Eq. (7) it can be checked whether all fixed payment constraints are satisfied.

$$\left(\sum_{t=1}^{n}\mathbf{E}_{tt}\mathbf{K}\mathbf{E}_{tt}\right)\vec{\mathbf{i}}=\vec{0} \tag{8}$$

5.1.2 Formulation and Solution of Residual Problems

If one of these two steps described above is not satisfied, Eq. (3) yields the residual problem. The initial problem matrix $\mathbf{P}^{j}$ and the inequality vector $\vec{\mathbf{u}}^{T}$ are not altered and can be used for the next partial solution process step.

5.2. Model under Risk

The model under risk distinguishes itself from the model under uncertainty by the introduction of probabilities of occurrence for each scenario. Thus, risk is captured in a discrete function. There is no separation between systematic and unsystematic risk [3]. The focus is again to ensure minimum cash inflows or maximum cash outflows, i.e. the shortfall risk remains the center of interest. Other risk parameters such as beta, volatility, residual volatility, correlation coefficient, tracking error are at least not covered in the constraints. Introducing different scenarios into the consulting and solution process marks a significant improvement compared to the status quo in practical financial planning consulting, scenarios without scenario probabilities will not suffice for a number of financing and especially investment problems.

From the perspective of the customer inequality constraints (Case IV) may be too restrictive since a payment *must not* fall below a specified value. To make sure that this specified value is reached at all costs, the customer may have to sacrifice a lot of potential return. Especially in the context of financial planning services, the used “best” and “worst” scenarios are often very unlikely compared to the “average” scenario, since they are usually based on historical data and mark the worst and best possible outcome over a couple of years or even decades. In addition, generally speaking at least subjective probabilities for scenarios can be obtained from historical data for most traded securities. From the perspective of the solution and decision process, all relevant information that is accessible (without prohibitive costs) should be included in the process to improve the quality of the decision.

5.2.1 The Financial Problem

The solution process is more difficult compared to the models under certainty and uncertainty. In contrast to the constraints of Case I to IV a *probability constraint* can not be formalized using linear equations or inequalities because it does not address a specific cash flow at one point in time t but a discrete random variable characterized by all scenario specific cash flows at one point in time t and the probabilities of the scenarios. Thus, the solution process considering probability constraints could not be performed solely by matrix algebra and another assumption is necessary.

(AD) Distribution function and scenario probabilities: The payment at time t within a (global) solution s^a is a discrete probability variable denoted by S_t^a. The corresponding distribution function is denoted by $\mathrm{F}_t^a(x)$. Let w^j denote the probability of occurrence of scenario j, with $\sum_j w^j = 1;\ w^j \geq 0 \quad \forall j$. This probability is assumed to be constant in time and independent of all partial solutions s^{al} and all other solution alternatives.

To capture cases that are similar to the one described in Example 3, another two cases have to be introduced:

- *Desired payment is a maximum cash outflow with a maximal probability* (Case Va): If v_t denotes the desired maximum cash outflow at time t with the maximal probability w_t^v, then all solutions s^a are feasible if and only if $W\left(S_t^a \leq v_t\right) \leq w_t^v \Leftrightarrow \mathrm{F}_t^a\left(v_t\right) \leq w_t^v$. $W\left(S_t^a \leq v_t\right)$ denotes the probability that S_t^a yields a value that is equal to or below v_t. Even though probability constraints are checked without using matrix algebra, the coefficients of the problem matrix and the problem vector still have to be set to zero for further calculations, thus
$$\begin{aligned} P_{ti}^j &= 0 \qquad && \text{for } i = 1, \ldots, n;\ j = 1, \ldots, m \\ p_t^j &= 0 && \text{for } j = 1, \ldots, m \end{aligned}$$
Rearranging Eq. (1) yields $0 s_t^{jal} = 0$, which is always true.
- *Desired payment is a minimum cash inflow with a minimal probability* (Case Vb): If v_t denotes the desired minimum cash inflow at time t with the minimal probability w_t^{v*}, then all solutions s^a are feasible if and only if $W\left(S_t^a > v_t\right) \geq w_t^{v*} \Leftrightarrow \mathrm{F}_t^a\left(v_t\right) \leq \underbrace{1 - w_t^{v*}}_{w_t^v}$. Obviously, Case Vb can be transformed into a formulation analogously to Case Va. Analogously to Case Va, the coefficients of the problem

matrix and the problem vector are set to zero.

$$P_{ti}^{j} = 0 \qquad \text{for } i = 1,\ldots,n;\ j = 1,\ldots,m$$
$$p_{t}^{j} = 0 \qquad \text{for } j = 1,\ldots,m$$

Rearranging Eq. (1) yields $0 s_t^{jal} = 0$, which is always true.

To check a solution s^a on feasibility with respect to a formulated probability constraint at a time t, first the distribution function $\mathrm{F}_t^a(x)$ has to be calculated. Solution s^a comprises all partial solutions s^{al} that have been integrated in s^a so far on the way to find a feasible solution after l partial solution process steps. A separated calculation for partial solutions, like in Sec. 4.2 and Sec. 5.2 does not suffice here anymore.

Each solution alternative s_t^a at time t is characterized by its payments s_t^{ja} in the various scenarios j and the respective probabilities of occurrence w^j. Summarizing the payments and the respective probabilities into a tuple, a solution for time t (the discrete probability variable) can be written as

$$S_t^a = \left[\left(s_t^{1a}; w^1\right) \quad \left(s_t^{2a}; w^2\right) \quad \ldots \quad \left(s_t^{ma}; w^m\right)\right] \tag{9}$$

To calculate the distribution function, first, the row of tuples has to be sorted ascending dependent on the value of the payment s_t^{ja}. The respective sorting function is denoted by Θ. After the sorting, the resulting tuples have the form $\left(s_{t,c}^a; w_{t,c}; j_{t,c}\right)$, where c denotes the rank among the tuples after the sorting took place and $j_{t,c}$ denotes the rank according to the scenarios before sorting. The coefficient t in $w_{t,c}$ reflects for which point in time the sorting took place.

$$\Theta\left[\left(s_t^{1a}; w^1\right) \quad \left(s_t^{2a}; w^2\right) \quad \ldots \quad \left(s_t^{ma}; w^m\right)\right] = \left[\left(s_{t,1}^a; w_{t,1}; j_{t,1}\right) \quad \ldots \quad \left(s_{t,m}^a; w_{t,m}; j_{t,m}\right)\right] \tag{10}$$

Having sorted the tuples, now an accumulation of the probabilities is necessary to get the distribution function. This operation is denoted by Φ.

Apparently, the constraint $\mathrm{F}_t^a(v_t) \le w_t^v$ is satisfied if point $\left(v_t; w_t^v\right)$ is located *on or above* the distribution function. To check whether the probability constraints are satisfied at time t the first tuple $\left(s_t^*; w_t^*; j_t^*\right)$ (denoted *critical tuple* in the following) has to be considered where the cumulated probability is above w_t^v. Thus, a condition of the form $\mathrm{F}_t^a(x) \le w_t^v$ is satisfied if and only if $x < s_t^*$. That is, for $\mathrm{F}_t^a(v_t) \le w_t^v$ to hold, the following statement has to be true.

$$v_t < s_t^* \Leftrightarrow s_t^* - v_t > 0 \tag{11}$$

Like in the simpler cases mentioned above, there may remain residual problems to be solved. How can a residual problem formally be described?

5.2.2 Formulation and Solution of Residual Problems

If the condition $s_t^* - v_t > 0$ (Eq. 11) is not true, this is equivalent to the statement that the solution so far provides for a payment that is too low in scenario j_t^* at time *t*. Therefore, for another partial solution $s^{ja(l+1)}$ at time *t* in scenario j_t^* the following condition – ε being some marginal value – has to be true:

$$s_t^{j_t^* a(l+1)} > -\left(s_t^* - v_t\right) \Leftrightarrow s_t^{j_t^* a(l+1)} \ge v_t - s_t^* + \varepsilon \tag{12}$$

Apparently, Eq. (12) corresponds to Case IV and the constraints formulated there. However, in contrast to Case IV the constraint for a minimum cash inflow and a maximum cash outflow is limited to a specific scenario here. Therefore, scenario specific problem matrices $\mathbf{P}^{jal}$ have to be introduced that are dependent not only on the scenario but also on the solution alternative *a* and the partial solution process step *l*. The integration of a residual problem into the scenario specific problem matrix and problem vector is accomplished by an *adaptation matrix* $\mathbf{A}^{jal}$ and *adaptation vector* $\vec{\mathbf{a}}^{jal}$.

- For each point in time *t* without a probability constraint and for each point in time *t* with a *satisfied* probability constraint the elements of the adaptation matrix $\mathbf{A}^{jal}$ and adaptation vector $\vec{\mathbf{a}}^{jal}$ are set to zero.

  $A_{ti}^{jal} = 0\,;\, a_t^{jal} = 0 \quad \forall i, j$

- For each point in time t with a probability constraint that is not satisfied, the elements of the adaptation matrix $\mathbf{A}^{jal}$ and adaptation vector $\vec{\mathbf{a}}^{jal}$ have to be altered according to the following rules

$$A_{ti}^{j_t^* al} = -1; \qquad A_{ti}^{j_t^* al} = 0 \quad \forall i \neq t; \qquad A_{ti}^{jal} = 0 \quad \forall j \neq j_t^*, i$$
$$a_t^{j_t^* al} = v_t - s_t^* + \varepsilon; \qquad a_t^{jal} = 0 \quad \forall j \neq j_t^*$$

Thus, the residual problem vector can be calculated as

$$\vec{\mathbf{p}}^{ja(l+1)} = \mathbf{P}^{jal}\vec{\mathbf{s}}^{jal} + \vec{\mathbf{p}}^{jal} + \vec{\mathbf{a}}^{jal} \tag{13}$$

and the corresponding adapted problem matrix as

$$\mathbf{P}^{ja(l+1)} = \mathbf{P}^{j} + \mathbf{A}^{jal} \tag{14}$$

Note that in Eq. (14) it is always the initial problem matrix $\mathbf{P}^j$ that is used to determine the problem matrix for the solution step (l+1). In contrast to Sec. 4.3 and Sec. 5.1.2 it is not sufficient here to check whether another partial solution just satisfies the constraints of the residual problem. Instead, it is inevitable to check the constraints also based on the complete aggregated solution, since the last integrated partial solution may alter the ranking of the tuples in Eq. (10) and thus may yield a different result based on Eq. (11) (See Appendix A for a detailed example).

So far, just the conditions to check a probability constraint have been discussed in this section. However, there may also be desired payment streams in a setting with scenarios and a probability distribution on these scenarios that correspond to the cases I to IV. To check a solution not only on the probability but on all constraints presented above, the following conditions have to be satisfied in order to call a solution a feasible solution.

- Check equality and inequality constraints:
  - Step 1: Check inequality constraints of the (residual) problem using the last partial solution s^{al}.
  - Step 2: Check equality constraints of the (residual) problem using the last partial solution s^{al}.

- Check probability constraint: Calculate the distribution functions of solution s^a for each necessary scenario *j* and point in time *t*.

If and only if both checks are satisfied with respect to the last partial solution s^{al} *and* the complete solution s^a, the solution is a feasible solution s^a.

5.2.3 Transformation of Probability Constraints

As briefly mentioned above, the presented procedure to deal with probability constraints has two major disadvantages. First, the (complete) solution s^a and its distribution function have to be calculated in each solution step proceeded by the check of the probability constraint(s). This increases the computing time. Second, residual problems resulting from unfulfilled probability constraint(s) are not completely described: further partial solutions may be feasible to the residual problem formulation, but the aggregated solution is unfeasible to the probability constraint. If the control system (cf. Sec. 3) triggers that no further locally optimized partial solutions shall be included, but the residual problem has to be solved (in order to generate a feasible solution), the decision system will not be able to accurately “find” a feasible partial solution by analyzing the payment structure of available partial solutions. To address these disadvantages, an innovative transformation of probability constraints into scenario specific minimum payment constraints is introduced in the following. The transformation consists of four steps:

1) Calculate all *m*! possible tuple orders (permutations) which may result after sorting the tuples for *m* scenarios and accumulate the probabilities to get the distribution functions. E.g. in case of three scenarios 3! = 6 different tuple orders (permutations) $X_e \in \{X_1;...;X_{M!}\}$ are possible.
2) Identify the critical tuple for each permutation based on the accumulated probabilities.
3) From the ranking of the tuples and the critical tuple of each permutation a set of scenario specific minimum payment constraints can be derived (*permutation constraints*), whereas a solution fulfilling

a set of constraints is feasible. E.g. in case of three scenarios 3! = 6 permutation constraints can be formulated.

4) Delete all double and unnecessarily restrictive permutation constraints.

The result is a disjunction of permutation constraints whereas each consists of a conjunction of scenario specific minimum payment constraints, i.e. it is sufficient for a solution to satisfy one permutation constraint to be feasible. Step 1) and 2) are illustrated in the following example.

Example 6: *Three scenarios exist with the probabilities* $w^1 = 0{,}25$, $w^2 = 0{,}6$ *and* $w^3 = 0{,}15$. *The probability constraint for point in time t = 3 can be written as* $\left(v_3 = 22; w_3^v = 0.1\right)$. *Depending on the payments of a solution in the different scenarios* s_t^{ja} *, the following tuple orders with the corresponding critical tuples can occur (permutations):*

	Tuple of Permutation X_e	*Critical Tuple*
X_1	$\left[\left(s_{3,1}^a; 0{,}25; 1\right) \quad \left(s_{3,2}^a; 0{,}85; 2\right) \quad \left(s_{3,3}^a; 1; 3\right)\right]_1 \overset{(v_3=22; w_3^v=0{,}1)}{\Rightarrow}$	$\left(s_t^*; w_t^*; j_t^*\right)_1 = \left(s_{3,1}^a; 0{,}25; 1\right)$
X_2	$\left[\left(s_{3,1}^a; 0{,}25; 1\right) \quad \left(s_{3,2}^a; 0{,}4; 3\right) \quad \left(s_{3,3}^a; 1; 2\right)\right]_2 \overset{(v_3=22; w_3^v=0{,}1)}{\Rightarrow}$	$\left(s_t^*; w_t^*; j_t^*\right)_2 = \left(s_{3,1}^a; 0{,}25; 1\right)$
X_3	$\left[\left(s_{3,1}^a; 0{,}6; 2\right) \quad \left(s_{3,2}^a; 0{,}85; 1\right) \quad \left(s_{3,3}^a; 1; 3\right)\right]_3 \overset{(v_3=22; w_3^v=0{,}1)}{\Rightarrow}$	$\left(s_t^*; w_t^*; j_t^*\right)_2 = \left(s_{3,1}^a; 0{,}25; 1\right)$
X_4	$\left[\left(s_{3,1}^a; 0{,}6; 2\right) \quad \left(s_{3,2}^a; 0{,}75; 3\right) \quad \left(s_{3,3}^a; 1; 1\right)\right]_4 \overset{(v_3=22; w_3^v=0{,}1)}{\Rightarrow}$	$\left(s_t^*; w_t^*; j_t^*\right)_4 = \left(s_{3,1}^a; 0{,}6; 2\right)$
X_5	$\left[\left(s_{3,1}^a; 0{,}15; 3\right) \quad \left(s_{3,2}^a; 0{,}4; 1\right) \quad \left(s_{3,3}^a; 1; 2\right)\right]_5 \overset{(v_3=22; w_3^v=0{,}1)}{\Rightarrow}$	$\left(s_t^*; w_t^*; j_t^*\right)_2 = \left(s_{3,1}^a; 0{,}25; 1\right)$
X_6	$\left[\left(s_{3,1}^a; 0{,}15; 3\right) \quad \left(s_{3,2}^a; 0{,}75; 2\right) \quad \left(s_{3,3}^a; 1; 1\right)\right]_6 \overset{(v_3=22; w_3^v=0{,}1)}{\Rightarrow}$	$\left(s_t^*; w_t^*; j_t^*\right)_2 = \left(s_{3,1}^a; 0{,}25; 1\right)$

After sorting the payments and cumulating the probabilities (step 2), the ranked payments of a specific permutation $\left[\left(s_{t,1}^a; w_{t,1}^{'}; j_{t,1}\right) \quad \ldots \quad \left(s_{t,c*}^a; w_{t,c*}^{'}; j_{t,c*}\right) \quad \ldots \quad \left(s_{t,m}^a; w_{t,m}^{'}; j_{t,m}\right)\right]$ fulfill

$s^a_{t,1} \leq ... \leq s^a_{t,c*} \leq ... \leq s^a_{t,m}$, wherein c^* denotes the rank of the critical tuple. If $v_t + \varepsilon \leq s^a_{t,c*}$ is true, the solution is feasible and

$$v_t + \varepsilon \leq s^a_{t,c*} \leq ... \leq s^a_{t,m} \tag{15}$$

is also true.

Based on this analysis of a specific permutation, we can now formulate a set (conjunction) of constraints for a solution to be feasible (step 3):

$$s^a_{t,c} \geq v_t + \varepsilon \quad \forall c \geq c* \quad \Leftrightarrow \quad s^a_{t,c*} \geq v_t + \varepsilon \quad \wedge \quad s^a_{t,c*+1} \geq v_t + \varepsilon \quad \wedge ... \wedge \quad s^a_{t,m} \geq v_t + \varepsilon\,. \tag{16}$$

It is not necessary to demand Eq. (16) to be true or to specify constraints for the payments $s_t^{j_{t,1}a}; s_t^{j_{t,2}a};...; s_t^{j_{t,c*-1}a}$ as any change in the ranking of the scenario tuples before or after the critical tuple will not destroy the feasibility of the solution if Eq. (15) holds. As Eq. (15) was derived from the analysis of a specific permutation X_e, the resulting set of scenario specific minimum payment constraints is denoted permutation constraint Z_e. If identical sets of minimum payment constraints result from different permutations or if permutation constraints are more restrictive than others[13], these can be abandoned (step 4). These permutation constraints do not offer additional useful information about the required structure of a feasible solution. Finally, a solution is feasible if it fulfills (at least) one remaining permutation constraint. Thus, the probability constraint was transformed into scenario specific minimum payment constraint, the problem is completely described and a major disadvantage of probability constraints was solved. But a new question arises: When shall the system compute the transformation – upfront, i.e. before the start of the heuristic, or later?

In case only one permutation constraint remains it is obviously advisable to transform the probability constraint upfront: the time to compute distribution functions can be saved and the checks for feasibility of (partial) solutions are faster. If two or more permutation constraints remain, the problem can be split into several problem formulations each including one permutation constraint. Feasible solutions

[13] E.g. a permutation constraint includes the same but also additional minimum payment constraints than another less restrictive permutation constraint.

shall be concurrently computed for all these problem formulations which increase the computing time. (Note that it is not advisable to focus on a subset of problem formulations as each comprises more restrictive minimum payment constraints than the original probability constraint. Feasible solutions may be unjustifiably declared as unfeasible and thus are lost.) In this situation it may be preferable to transform the probability constraint not before the heuristic commands that a feasible solution shall result after the next addition of a partial solution. Until this instant the check of feasibility is accomplished as described at the end of Sec. 5.2.2. To include the scenario specific minimum payment constraints into the (residual) problem formulation the *adaptation matrixes* $\mathbf{A}^{jal}$ and *adaptation vectors* $\bar{\mathbf{a}}^{jal}$ introduced in Sec. 5.2.2 can be used accordingly.

It has been shown formally how feasible solutions can be generated if fixed, arbitrary, minimum and maximum payments as well as minimum payments with a minimal probability and maximum payments with a maximal probability are required. As described in Sec. 3, this step of the overall problem solution process is followed by the valuation of the feasible solutions applying a valuation function and a selection of the solutions to be presented (for different evaluation functions in this context see e.g. [17]).

6. Discussion and Limitations of the Model

The presented model contributes to an improvement in the quality of the consultation process in at least two ways: First, due to the obligatory starting point of the process with the financial problem of the customer, a product centric view can be circumvented. Second, the model fosters the integration of already existing local optimization knowledge. Thus, applications that have already been developed for a local optimization can still be used if the implementation provides for a sufficient modularization.

Talking about the convergence towards a superior solution, so far the model has only been implemented in a simpler form in comparison to the model proposed above. Thus, no empirical tests could be carried out, whether a convergence can be expected in the case of uncertainty or risk. However, there are reasons for hope that the hybrid recognition and search process converges towards qualitatively good solutions. First, combination knowledge that is already available can be incorporated in the solution process. Thus, at least standard solutions that are widely offered today will be generated and in so far the model will *at least* ensure the status quo of the quality of recommendations in the financial services sector today. Second, in the ALLFIWIB project already mentioned above ([4], [7]) it could be shown in a prototypical implementation that superior solutions are generated and can be expected using this approach - at least under certainty.

Besides the question of convergence, there are another three issues that limit the above model to some extent: risk representation, dependencies between partial solutions and constant marginal tax rate.

First, the representation of risk can be criticized. Especially the constraints that can be formulated by the customer concerning minimum cash inflows or maximum cash outflows – eventually with a specific probability – just capture shortfall risks but do not take into account any chances. Applying an appropriate evaluation function, this situation can be relaxed. If the evaluation function takes into account also chances as opposed to just focusing on the downside risk, a well balanced decision can be safeguarded. In addition, the probabilities of occurrence were assumed to be constant in time, across discrete scenarios and across all solutions. This may be in most instances an oversimplification, however, the introduction of time-specific probabilities into the model would not pose a big difficulty. Knowledge about correlation of two or more financial products that may be used in an optimization process can be considered in two ways. *Between* two partial solutions a low correlation is represented implicitly if one partial solution has high (low) payments in scenarios where the other partial solution has low (high) payments. Second, correlation can be accounted for explicitly *within* a partial solution, e.g. if a partial solution is a portfolio of securities optimized with Markowitz’s portfolio theory.

An implicit assumption of the model is the independency of the cash flows between partial solutions, i.e. the cash flow of one solution is independent from the decision whether other partial solutions are added to form a solution. E.g. in case of a loan this might not be true as the purchase of a partial solution “life insurance” reduces the credit risk, which subsequently has an influence on the interest rate and finally on the cash flow of the partial solution “loan”.

Analogously - depending on the tax regime of the country where the investor assessed - the assumed constant marginal tax rate may in a number of cases constitute an oversimplification. In a progressive tax regime, it is well imaginable that a partial solution generates such high tax deductible amounts that the marginal tax rate is lowered after the integration of this partial solution. However, this would most likely have effects on all partial solutions already integrated and also on the efficiency of the initial portfolio.

7. Conclusion

A model has been presented that allows for the inclusion of uncertainty and risk into the formulation of financial problems by the customer as well as in the solution process, i.e. intelligently bundling financial products to form a superior solution for a specific customer problem. The presented formal model is just a first step to better incorporate risk in the financial planning process and facilitate the use of information technology for the solution generation process. Especially customer segments with comparably structured problems and a limited problem domain such as the Affluent segment may benefit substantially by a DSS enabled financial planning concerning the solution generation process. Today, this segment cannot be serviced appropriately due to the prohibitive high costs, but tomorrow supported by adequate applications in combination with well-trained staff this may become a sustainable competitive advantage.

Moreover, a major innovation in this contribution is the proposed transformation of probability constraints into scenario specific minimum payment constraints, which is not only applicable in the domain of financial planning. This transformation and solution algorithm can be extended to the class of decision problems where scenarios (and scenario specific probabilities) are used to capture risk and constraints that require (deterministic) minimum or maximum outcomes with a specified probability.

References

[1] K.J. Arrow, Alternative Approaches to the Theory of Choice in Risk-Taking Situations, Econometrica 19 (1951) pp. 404-437.

[2] M. Benaroch and V. Dhar, Controlling the complexity of investment decisions using qualitative reasoning techniques, Decision Support Systems 15 (2) (1995), pp. 115-132.

[3] R.A. Brealey and S.C. Myers, Principles of Corporate Finance (McGraw-Hill, New York, 1996).

[4] H.U. Buhl, H.J. König, A. Will, ALLFIWIB: Distributed Knowledge Based Systems for Customer Support in Financial Services, in: A. Karmann, K. Mosler, M. Schader, G. Uebe, eds., Operations Research '92 (Physica, Heidelberg, 1993) pp. 537-540.

[5] H.U. Buhl, D. Kundisch, W. Steck, A. Leinfelder, IT-Enabled Sophistication Banking, in: H. Werthner, M. Bichler, H. Mahrer, eds., Proceedings of the 8th European Conference on Information Systems ECIS 2000 (Vienna, 2000) pp. 789-795.

[6] B. Elbracht-Hülseweh, Problemlösungsverhalten und Problemlösungsmethoden bei schlechtstrukturierten Problemen (Brockmeyer, Bochum, 1985).

[7] U. Einsfeld, M. Roemer, P. Roßbach, K. Sandbiller, A. Will, ALLFIWIB: Supporting the Process of Customer Consulting in Financial Services with Distributed Knowledge Based Systems, in: W. König, K. Kurbel, P. Mertens, D. Preßmar, eds., Distributed Information Systems in Business, (Springer, Berlin, 1996) pp. 213-228.

[8] H.B. Eom and S.M. Lee, A Large-Scale Goal Programming Model-Based Decision Support for Formulating Global Financing Strategy, Information & Management 12(1) (1987) pp. 33-45.

[9] M.C. Er, Decision Support Systems: A Summary, Problems, and Future Trends, Decision Support Systems 4(3) (1988) pp. 355-364.

[10] F. Gardin, R. Power, E. Martinelli, Liquidity management with fuzzy qualitative constraints, Decision Support Systems 15(2) (1995) pp. 147-157.

[11] W. Gaul, Financial Planning via Stochastic Programming: A Stochastic Flows-With-gains Approach, in: G. Bamberg, ed., Risk and capital (Springer, Berlin, 1983) pp. 227-243.

[12] H. Hax, Entscheidungsmodelle in der Unternehmung. Einführung in Operations Research, (Rowohlt, Reinbeck, 1974).

[13] R.L. Hayen, Applying decision support systems to small business financial planning, Journal of Small Business Management 20(3) (1982) pp. 35-47.

[14] B. Hayes-Roth, A Blackboard Architecture for Control, Artificial Intelligence 26 (1985), pp. 251-321.

[15] M. Klein and T. Villedieu, A KB/DSS for Financial Planning, Engineering Costs and Production Economics 12 (1-4) (1987) pp. 219-228.

[16] F.H. Knight, Risk, Uncertainty and Profit (University of Chicago Press, Chicago, 1921).

[17] D. Kundisch, New Strategies for Financial Services Firms (Physica, Heidelberg, 2003).

[18] D. Kundisch and J. Dzienziol, Formalization of Financial Problems and Solutions under Risk as an Essential Requirement for IT-based Financial Planning, Proceedings of the 36th Annual Hawaii International Conference on System Sciences (IEEE Computer Society Press, Los Alamitos, Hawaii, 2003).

[19] D.B. Lenat, The role of heuristics in learning by discovery: Three case studies, in: R.S. Michalski, J.G. Carbonell, T.M. Mitchell, eds., Machine Learning: An Artificial Intelligence Approach (Tioga Presse, Palo Alto, 1983) pp. 243-306.

[20] H. Locarek and C. Preuss, DSFINANCE – Decision Support for Financial Planning, in: W. Gaul, M. Schader , eds., Knowledge, Data and Computer-Assisted Decisions (Springer, Berlin, 1990) pp. 337-347.

[21] D. Maringer, Portfolio management with heuristic optimization (Dordrecht, Springer, 2005).

[22] H. Markowitz, Portfolio Selection: Efficient Diversification of Investments (John Wiley & Sons, New York, 1959).

[23] B.J. McCabe and C.P. Boinske, Wealth Planning Under Uncertainty, Journal of Financial Planning 13(3) (2000) pp. 84-96.

[24] G. P. Moynihan, P. Purushothaman, R. W. McLeod, W. G. Nichols, DSSALM: A decision support system for asset and liability management, Decision Support Systems 33(1) (2002) pp. 23-38.

[25] A. Palm-dos-Reis and F. Zahedi, Designing personalized intelligent financial decision support systems, Decision Support Systems 26 (1999) pp. 31-47.

[26] D.J. Power and R. Sharda, Model-driven decision support systems: Concepts and research directions, Decision Support Systems, In Press, Corrected Proof, Available online 20 July 2005.

[27] K. Sandbiller, C. Weinhardt, A. Will, “Cooperative Agents Solving Financial Problems: A Scenario”, Contribution to the Meeting of the working group “Verteilte Künstliche Intelligenz” of the GI, 17. – 18.12.1992.

[28] J.J. Siegel, Stocks for the long run (McGraw-Hill, New York, 1998).

[29] S. Wang and Y. Xia, Portfolio Selection and Asset Pricing (Springer, Berlin, 2002).

[30] A. Will, Die Erstellung von Allfinanzprodukten: Produktgestaltung und verteiltes Problemlösen (Deutscher Universitäts-Verlag, Wiesbaden, 1995).

[31] J.W. Wilkinson, Effective Financial Planning Model, Journal of Systems Management, 38(6) (1987), pp. 6-11.

[32] H.P. Wolfersberger, Individualisierung von Finanzdienstleistungen - die optimale Disagiovariante eines Festdarlehens, in: Zeitschrift für Betriebswirtschaft 72(6) (2002) pp. 573-591.

[33] W. Zhang, R. Dechter, R. E. Korf, eds, Heuristic search in artificial intelligence, Artificial Intelligence 129(1-2) (2001).

Appendix A: Example for Sec. 5.1

The probability constraint of Mr. Smith in Example 3 – to receive more than 22,000 Euro after two years (v_3 = 22) with a probability of at least 90% ($w_3^{v^*} = 0.9$) – corresponds to type Vb and can formally be written as $W\left(S_3^a > 22\right) \geq 0.9 \Leftrightarrow \mathrm{F}_3^a(22) \leq \underbrace{1-0.9}_{w_3^v} = 0.1$.

Mr. Smith is offered a funds investing in European stocks as a first (partial) solution (l = 1) within a solution alternative s^{21} (a = 2). The funds is expected to yield 26,000 Euro with 25% probability in the "best" (w_1 = 0.25), 23,000 Euro with 60% probability in the "average" (w_2 = 0.6), and 18,000 Euro with 15% probability in the "worst" scenario (w_3 = 0.15) in 2 years. Probability variable S_3^2 *at time t = 3 can be written as* $\left[\left(s_3^{121} = 26;\, w^1 = 0.25\right)\left(s_3^{221} = 24;\, w^2 = 0.6\right)\left(s_3^{321} = 18;\, w^3 = 0.15\right)\right]$.

Sorting this expression and cumulating the probabilities yields

$$\Phi\left[(18;\,0.15;\,3)(24;\,0.6;\,2)(26;\,0.25;\,1)\right] = \left[(18;\,0.15;\,3)(24;\,0.75;\,2)(26;\,1;\,1)\right].$$

This offered solution has to be checked on the probability constraint of Mr. Smith from Example 3. The relevant tuple is $\left(s_3^* = 18; w_3^* = 0.15; j_3^* = 3\right)$ *and the probability constraint is* $\left(v_3 = 22; w_3^v = 0.1\right)$ *at time t = 3. The point* $\left(v_3 = 22; w_3^v = 0.1\right)$*, representing the probability constraint, is obviously located below the distribution function* $\mathrm{F}_3^2(x)$*. Thus, the probability constraint is not satisfied.*

Apparently, another partial solution (l = 2) s^{22} has to provide in the "worst" scenario a cash inflow after two years (t = 3) that is greater than 4,000 Euro (v_3 = 4), i.e. $s_3^{322} > 4 \Leftrightarrow s_3^{322} \geq 4 + \varepsilon$ [14]*. The constraints concerning the two fixed payments today (t = 1) and in one year (t = 2) were satisfied. To formally determine the residual problem, first the adaptation matrices* $\boldsymbol{A}^{j21}$ *and vectors* $\vec{\mathbf{a}}^{j21}$ *have to be determined.*

$$\mathbf{A}^{121} = \mathbf{A}^{221} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \mathbf{A}^{321} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}; \vec{\mathbf{a}}^{121} = \vec{\mathbf{a}}^{221} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}; \vec{\mathbf{a}}^{331} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$$

[14] For reasons of clarity the marginal variable is not shown in the vectors and matrices below but is only used at the end of the calculation to check whether the constraint is satisfied.

Thus, the problem matrices $\mathbf{P}1$ *and* $\mathbf{P}2$ *equal the initial problem matrix (see Example 5), whereas* $\mathbf{P}3$ *is altered.*

$$\mathbf{P}^1 + \mathbf{A}^{121} = \mathbf{P}^2 + \mathbf{A}^{221} = \mathbf{P}^{122} = \mathbf{P}^{222} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \mathbf{P}^j,$$

$$\mathbf{P}^{322} = \mathbf{P}^3 + \mathbf{A}^{321} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

The problem vectors in the "best" and "average" scenario for the residual problem are

$$\vec{\mathbf{p}}^{122} = \mathbf{P}^1\vec{\mathbf{s}}^{121} + \vec{\mathbf{p}}^1 + \vec{\mathbf{a}}^{121} = \vec{\mathbf{p}}^{222} = \mathbf{P}^2\vec{\mathbf{s}}^{221} + \vec{\mathbf{p}}^2 + \vec{\mathbf{a}}^{221} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} -10 \\ -10 \\ 0 \end{pmatrix} + \begin{pmatrix} -10 \\ -10 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \vec{0}.$$

Obviously, the constraints concerning the fixed payments are satisfied in these scenarios. For the problem vector in the "worst" scenario Eq. (19) yields

$$\vec{\mathbf{p}}^{322} = \mathbf{P}^3\vec{\mathbf{s}}^{321} + \vec{\mathbf{p}}^3 + \vec{\mathbf{a}}^{322} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} -10 \\ -10 \\ 0 \end{pmatrix} + \begin{pmatrix} -10 \\ -10 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}.$$ *A feasible solution for the residual problem has to satisfy Eq. (6) and Eq. (8). A possible partial solution* s^{22} $(l = 2)$ *for this residual problem is to sell a futures contract with a maturity of two years*[15] *and the following payment streams*

$$s^{22} = \left\{ \vec{\mathbf{s}}^{122} = \begin{pmatrix} 0 \\ 0 \\ -5 \end{pmatrix}; \vec{\mathbf{s}}^{222} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}; \vec{\mathbf{s}}^{322} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} \right\}.$$ *It can be shown that this partial solution satisfies Eq. (6) as well as Eq. (8) and solves the residual problem. However, this does not need to mean in turn that also a global solution has been found as the residual problem does not describe the necessary payment structure completely. The probability constraint has to be checked using the (global) solution* s^2. *The new probability variable* S_3^2 *of solution* s^2 *can be described as* $S_3^2 = [(21;0.25)(24;0.6)(23;0.15)]$. *Sorting*

[15] Abstracting form margin payments, clearing fees, etc., there are no real cash inflows or outflows before maturity associated with the purchase of a futures contract. On futures contracts see e.g. [3].

these tuples using Eq. (10) and accumulating the probabilities using Eq. (11) yields:

$\Phi\left(\Theta S_3^2\right) = [(21;0.25;1)(23;0.40;3)(24;1;2)]$.*The relevant tuple for the check on feasibility is (21;0.25;1).*

Apparently, $s_3^* - v_3 = 21 - 22 = -1 < \varepsilon$ *. Thus, the global solution does not satisfy the probability constraint and solution* s^2 *is an unfeasible solution.*

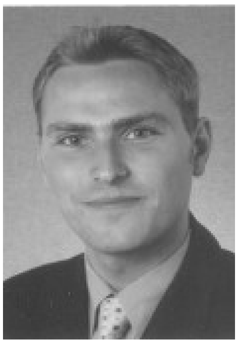
$$s^2 = s^{21} + s^{22} = \left\{ \vec{\mathbf{s}}^{12} = \begin{pmatrix} -10 \\ -10 \\ 21 \end{pmatrix}; \vec{\mathbf{s}}^{22} = \begin{pmatrix} -10 \\ -10 \\ 23 \end{pmatrix}; \vec{\mathbf{s}}^{32} = \begin{pmatrix} -10 \\ -10 \\ 24 \end{pmatrix} \right\}$$

Biographical Information

Mr. *Jochen Dzienziol* is a PhD student with the Department of Information Systems and Financial Engineering and project manager at the Competence Center IT & Financial Services at the University of Augsburg, Germany. He received a diploma in Business Administration as well as a Master's degree in Financial Management and Electronic Commerce from the University of Augsburg in 2002. His research interests include the customer lifetime value management in the financial services industry, decision support in financial planning and financial engineering.

Dr. *Dennis Kundisch* is an assistant professor with the Department of Information Systems and Financial Engineering and project manager at the Competence Center IT & Financial Services at the University of Augsburg, Germany. He holds a Master's degree in Business Administration from the University of Dayton and a diploma in Business Administration from the University of Augsburg. In 2002 he received his doctoral degree for his work on IT-enabled strategies and concepts in the financial services industry. Currently he is the coordinator of the Elite Graduate Program "Finance & Information Management" in the Elite Network of Bavaria. His research interests include strategies and concepts in electronic financial services, financial engineering, decision support in financial planning, and online brokerage.

Photographs



Jochen Dzienziol

Dr. Dennis Kundisch