



Discussion Paper

Bringing value-based Business Process Management to the Operational Process Level

by

Manuel Bolsinger¹

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¹ At the time of writing this paper, Manuel Bolsinger was a research assistant at the Research Center Finance & Information Management and the Department of Information Systems Engineering & Financial Management at the University of Augsburg.

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Manuel Bolsinger

Bolsinger, Manuel, Research Center Finance & Information Management, University of Augsburg, Universitaetsstrasse 12, 86159 Augsburg, Germany, manuel.bolsinger@fim-rc.de, +49 821 – 598 4885 (phone), +49 821 – 598 4899 (fax)

Abstract

For years, improving processes has been a prominent business priority for Chief Information Officers. As expressed by the popular saying, “If you can’t measure it, you can’t manage it,” process measures are an important instrument for managing processes and corresponding change projects. Companies have been using a value-based management approach since the 1990s in a constant endeavor to increase their value. Value-based business process management introduces value-based management principles to business process management and uses a risk-adjusted expected net present value as the process measure. However, existing analyses of this issue operate at a high (i.e., corporate) level, hampering the use of value-based business process management at an operational process level in both research and practice. Therefore, this paper proposes a valuation calculus that brings value-based business process management to the operational process level by showing how the risk-adjusted expected net present value of a process can be determined. We demonstrate that the valuation calculus provides insights into the theoretical foundations of processes and helps improve the calculation capabilities of an existing process-modeling tool.

Keywords: Value-Based Business Process management; Process Modeling; Process Measure; Net Present Value; Certainty Equivalent; Expected Value; Variance

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1 Introduction

Constant change in their economic, political, and social environments is forcing companies to strive for increased efficiency and more frequent innovation (Becker and Kahn 2005, p. 3), a situation in which the management and, in particular, the improvement of processes play a considerable role (González et al. 2010; Thome et al. 2011; van der Aalst 2013; vom Brocke et al. 2011a). One indicator of process improvement's prominent role is the fact that companies invest considerable amounts of money to develop their business process management (BPM) capabilities and realize improvement activities (Wolf and Harmon 2012). The volume of research on process improvement has also increased (Sidorova and Isik 2010, p. 572).

In their efforts to improve processes, researchers and practitioners alike must establish a basis on which it can be decided that an alternative (or "to-be") process is better than an existing (or "as-is") process. The instruments deemed appropriate for determining the extent to which a process alternative improves an existing process are called "process measures" (González et al. 2010; Tregear 2012; zur Muehlen and Shapiro 2010). When the value of a process measure of an alternative process is greater than that of an existing process, it might be reasonable to implement the alternative process and thus improve the existing process. However, there are many process measures, and, while the value of one measure may suggest a process improvement, the value of another may indicate the opposite. For example, the dimensions of time, cost, quality, and flexibility, often used to evaluate process improvement, comprise the so-called "devil's quadrangle" because, "in general, improving [a process] upon one dimension may have a weakening effect on another" (Reijers and Liman Mansar 2005, p. 294). Hence, process managers have to consider these complementary and competitive goal relations when determining whether an alternative process improves an existing process. In order to resolve potential conflicts among goals, process managers need integrated approaches that consolidate various goals into one overall goal, thus allowing them to make decisions based on that overall goal.

Value-based BPM introduces into BPM an overall goal in line with economic theory (Buhl et al. 2011). Value-based BPM applies value-based management principles to process decision-making and aims to increase company value from a long-term perspective (Ittner and Larcker 2001; Koller et al. 2010; Young and O'Byrne 2001), thus supporting process improvement from a monetary-centered view of BPM. Companies have been using value-based management since the 1990s in their constant endeavor to increase their value (Coenenberg and Salfeld 2007, p. 3). Almost two thirds of the 30 companies on the German stock index (DAX), representing Germany's major companies, explicitly state in their 2013 annual reports that they follow a value-based management approach. Moreover, the 2013 CIO agenda (Gartner 2013) identified "harvest value from business process changes" as one of their three performance profiles. Hence, value-based BPM not only provides an approach for integrating different goals but also takes on a business perspective by facilitating the overall goal of increasing company value, wherein a process' value contribution is determined by its risk-adjusted expected net present value, or "rNPV" (Bolsinger et al. 2011; Buhl et al. 2011). A process alternative should be implemented as an improvement whenever its rNPV is higher than that of the existing process.

However, although research suggests the transferability of value-based management to BPM, current studies operate at a high (i.e., corporate) level and do not show how the rNPV is to be calculated in detail, particularly with reference to a process' control flow, which is important to connect the corporate level with the operational level (Rotaru et al. 2011; vom Brocke et al. 2010). Furthermore, in the practice of BPM, modeling tools (e.g., IBM WebSphere Business

Modeler Advanced, Bonita Studio, TIBCO Business Studio, ibo Prometheus Klassik and Bizz Designer) cannot determine the rNPV and, thus, do not support value-based BPM. In order to substantiate value-based BPM from both theoretical and practical points of view, additional research capable of establishing the appropriate theoretical foundations is necessary (Vergidis et al. 2008).

This paper contributes to the literature *by providing a valuation calculus for determining the risk-adjusted expected net present value of a process*. After the valuation calculus is implemented, a process-modeling tool could calculate the rNPV for various process alternatives, from which a process manager could choose for a process improvement project. This functionality would provide a valuable asset for process managers (van Hee and Reijers 2000; Vergidis et al. 2008) and bring value-based management into the practice of BPM.

This paper, reflecting the design science research process presented in Peffers et al. (2008), is organized as follows. After *motivating the importance of the problem* in this section, Section 2 provides more background information about value-based BPM and positions it against other BPM approaches related to value-based BPM. Based on this theoretical background, we derive the requirements for the valuation calculus that *define its objectives* before discussing related work. In Section 3, we introduce a basic illustrative example to provide a better understanding of the issues raised in the subsequent sections. In Section 4, the valuation calculus (our *artifact*) is *designed* using a formal-deductive research approach (Meredith et al. 1989). In Section 5, we focus on the *evaluation* of the valuation calculus in an artificial setting (Sonnenberg and vom Brocke 2012; Venable et al. 2012). We then present a feature comparison, a comparison with a related artifact, and a *demonstration* of the feasibility of the artifact by solving an exemplary problem instance and by illustrating how the knowledge of the valuation calculus corrected the calculation logic of the process-modeling tool of the CubeFour company. Finally, the last section summarizes our results and provides an outlook for future study.

2 Theoretical Background

2.1 Value-based Business Process Management

The value-based BPM paradigm focuses on the value that a newly designed process or a change in an existing process contributes to a company (Buhl et al. 2011; vom Brocke et al. 2010). In doing so, value-based BPM introduces value-based management principles to BPM, thus motivating process-related decisions according to a well-established management approach. Before discussing value-based BPM in detail, we will first outline the principles of value-based management.

Value-based management aims to sustainably increase a company's value from a long-term perspective (Ittner and Larcker 2001; Koller et al. 2010; Young and O'Byrne 2001). It extends the shareholder value approach that traces back to Rappaport (1986) and was further advanced by Copeland et al. (1990) and Stewart and Stern (1991). Taking a long-term perspective, value-based management complies with the stakeholder value approach (Danielson et al. 2008), which is important for a less decision-making oriented perspective on value-based BPM (vom Brocke et al. 2009). For value-based management to be fully realized, all activities on all company levels must be aligned with the goal of maximizing company value (Coenenberg and Salfeld 2007). The same holds true for company processes: each process has to contribute to the value of the company, and a process should be changed only if its value contribution can be increased.

Following a value-based management in BPM requires that process decisions be based on cash flows, that the time value of money be considered, and that the risks associated with the cash flows be taken into account (Buhl et al. 2011), all of which support process improvement from a monetary-centered view of BPM. The risks arise because cash flows are uncertain; thus, cash flows are modeled as random variables. These cash flows originate from every execution of a process, each of which is executed not only a few times but several times within a given planning horizon. This cash flow structure is brought together into one quantity through the net present value (NPV). The NPV of a process is thus uncertain, which is why it is also modeled as a random variable, and builds the foundation of a value-based BPM. As described in Bolsinger et al. (2011), the NPV of a process is expressed as follows:

$$NPV = -I + \sum_{t=0}^T \frac{\sum_{j=1}^{n_t} CF_{P_j}}{(1+i)^t}, \quad (1)$$

where I denotes an initial process investment,
 $T + 1$ the number of periods that a process will be executed within a certain planning horizon,
 n_t the number of times a process is executed within a period t ,
 CF_{P_j} the process cash flow of the j th execution of process P , and
 i “the rate of interest which properly reflects the investor’s time value of money” (Hillier 1963, p. 447).

The initial process investment can be, for example, the cash outflow needed to design an alternative process or change to one. This investment is different for each process alternative and can be set to zero for the existing process when comparing process alternatives to the existing one.

As mentioned, NPV is an uncertain quantity because CF_P is uncertain. Therefore, comparing the $NPVs$ of different processes is difficult because no process (alternative) has a single value by which the best process (alternative) (i.e., that with the best NPV) may be determined. To comply with value-based management, value-based BPM uses the expected utility theory to determine a single value per process (alternative) by using the certainty equivalent Φ of NPV (Buhl et al. 2011; Copeland et al. 2005, p. 54). The certainty equivalent corresponds to the process’ contribution to company value and is (as mentioned) the $rNPV$. The certainty equivalent is expressed as follows:

$$\Phi = E[NPV] - \frac{\alpha}{2} Var[NPV], \quad (2)$$

where $E[NPV]$ denotes the expected value of NPV ,
 $Var[NPV]$ the variance of NPV , and
 α the risk aversion constant, representing the risk attitude of the decision maker (Freund 1956).

The expected value is used as a process measure to capture the expected return of a process, while the variance is used to measure the risk of a process. The expected value is adjusted by the risk, depending on the risk attitude of the decision maker. The adjustment of the expected value results in the risk-adjusted value the decision maker assigns to the process. Bamberg and Spremann (1981) show how it is possible to elicit the needed information from decision makers to determine their utility function and translate it into a value of α . Decision makers must be asked certain questions, from which the utility function is then determined. More about

preference elicitation for utility measurement can be found in works such as Abdellaoui et al. (2013), Andersen et al. (2008), Beer et al. (2013), Friedman and Savage (1948), Mosteller and Nogee (1951), and Swalm (1966). Another approach to determining α is the market price perspective, which uses the capital asset pricing model (CAPM). In this model, $\alpha/2$ is the market price of risk, which can be determined through the CAPM's so-called "price equation" (Kruschwitz and Husmann 2010). Kasanen and Trigeorgis (1994) show how α can be calculated within the CAPM and it is estimated using actual market data (the authors' parameter m corresponds to our α).

The result is an integrated risk/return decision function based on a theoretically well-founded method, which is also used to make decisions in other domains (Datar et al. 2001; Fridgen and Müller 2009; Gibbons 2005; Longley-Cook 1998; Sen and Raghu 2013; Zimmermann et al. 2008). The certainty equivalent is used to decide if a process alternative improves an existing process (see Figure 1).

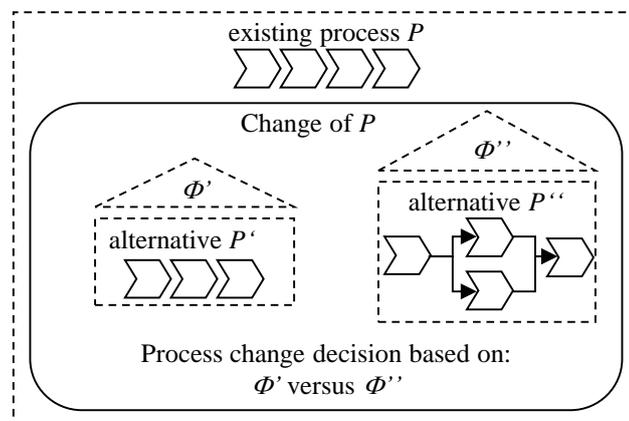


Figure 1 Process change decisions regarding process improvement

The merits and limitations of value-based BPM become clearer when positioned against related approaches such as goal-oriented BPM (Kueng and Kawalek 1997; Neiger and Churilov 2004a), value-focused BPM (Neiger and Churilov 2004b; Rotaru et al. 2011), value-driven BPM (Franz et al. 2011), and value-oriented BPM (vom Brocke et al. 2010).

Goal-oriented BPM demands that processes fulfill certain goals, which must be clearly stated in order to clarify what the process must achieve or avoid (Kueng and Kawalek 1997); the goals can be either functional (e.g., "sell insurance") or non-functional (e.g., low operational costs, short cycle time). Whatever goals are chosen, "the goal-oriented view of business process engineering dictates that business goals are the driving force for structuring and evaluating business processes" (Neiger and Churilov 2004a, p. 150). Thus, the goals provide the basis for evaluating how well a process is designed, but the process managers have to decide what those goals will be.

Value-focused BPM shows how value-based thinking (Keeney 1994) helps elicit essential goals from decision makers, facilitating goal-oriented BPM. In this context, values are "principles for evaluating the desirability of any possible alternative or consequence. They define all that you care about in a specific decision situation" (Keeney 1994, p. 33). Value-focused BPM shows how value-based thinking can substantiate the goals of a process and be incorporated into process modeling (Neiger and Churilov 2004b).

Value-driven BPM provides the values to which organizations aim when beginning a BPM initiative. These values consist of the core value "transparency" and the three value pairs

“efficiency-quality,” “agility-compliance,” and “integration-networking” (Franz et al. 2011). These values are suggested as BPM goals, each pair consisting of “two values that tend to be oppositional” (Franz et al. 2011, p. 6) therefore presenting conflicting goals. Thus, possible goals of goal-oriented BPM have been provided, but how to measure them or consolidate them into one overall goal and resolve their conflicts is not stated.

Finally, value-based and value-oriented BPM both have the goal of determining processes’ and process changes’ long-term business value (Buhl et al. 2011; vom Brocke et al. 2010), substantiating the goals of goal-oriented BPM. Both approaches are also based on capital budgeting methods. While, as discussed in vom Brocke et al. (2010), value-oriented BPM uses the Visualization of Financial Implications (Grob 1993) to value a process, value-based BPM, as illustrated in Buhl et al. (2011), uses the certainty equivalent method (Copeland et al. 2005, p. 54). Both methods are based on cash flows and consider the time value of money. The Visualization of Financial Implications provides in-depth insights into the payment structure of a process and can be used in a detailed analysis of processes from a financial perspective. The certainty equivalent method brings decision theory, in the form of the expected utility theory (Bernoulli 1954), into capital budgeting and represents a kind of semi-subjective valuation (Kruschwitz and Löffler 2003). This valuation considers a decision maker’s estimation of the utility of a financial value and allows the incorporation of the risk associated with that value as well as the risk attitude of the decision maker. Thus, while value-oriented BPM provides more detail about the payment structure, value-based BPM proposes an objective function that is “well-founded in terms of investment and decision theory” (Buhl et al. 2011, p. 170). Overall, both approaches are closely related and provide an important economic perspective to BPM, adding the well-founded, non-functional goals to goal-oriented BPM, as deemed necessary in Kueng and Kawalek (1997). As noted in vom Brocke et al. (2010), the value-oriented/value-based perspective has its limitations in that it does not necessarily consider other drivers for process improvement, such as compliance management. However, process improvement projects “in their essence present significant investments (Devaraj and Kohli 2001) to project sponsors who, ultimately, are interested in the return-on-investment from engaging in process re-design projects” (vom Brocke et al. 2010, p. 335). Hence, project sponsors are interested in the bottom line impact of their investment, thus focusing on the value-oriented/value-based perspective.

2.2 Requirements

We condense the remarks made so far regarding value-based BPM into the requirements below, which serve as our design objectives and the considerations we use to calculate the rNPV of a process; we also use the requirements when analyzing related studies in the next section:

- (R1) *Control flow*: Value-based BPM relies on a process’ rNPV as a process measure. To calculate the rNPV, the control flow of the process under consideration must be considered; this details how the corporate level is connected to the operational level because even a minor change in the control flow can result in a major change of the rNPV.
- (R2) *Cash flows*: The rNPV is based on the cash flows at the operational level.
- (R3) *Long-term perspective*: The rNPV does not consider only one period but can cope with a time horizon of several periods, incorporating a long-term perspective into value-based BPM and allowing the consideration of money’s time value.
- (R4) *Risk*: In value-based BPM, process risk is measured as the variance of its NPV, making it necessary to be able to calculate not only the NPV’s expected value but also its variance.

2.3 Related Work

This paper contributes to the value-based BPM literature, as described in Section 2.1, by attempting to connect the corporate level with the operational level by substantiating process rNPV calculation. We now review the relevant research in the BPM field that brings a value-oriented/value-based perspective to BPM. We discuss how this work addresses the requirements for value-based BPM outlined in Section 2.2. The overview on value orientation in BPM by Buhl et al. (2011) contains relevant papers. We briefly discuss the three that best fulfill the requirements: vom Brocke et al. (2010), Linderman et al. (2005), and Bai et al. (2007). We also discuss Buhl et al. (2011) because it not only surveys the literature but also contributes to economically well-founded BPM decisions. In addition to the works included in the overview on value orientation in BPM, we add others published after the overview appeared in order to include more recent research. These works are Bolsinger et al. (2011), Sampath and Wirsing (2011), and Wynn et al. (2013).

The work that best fulfills the requirements is vom Brocke et al. (2010), previously discussed in Section 2.1. The authors choose among process alternatives in order to improve a process on the basis of the (expected) terminal value of the investment and/or the return on investment (ROI). The terminal value considers cash flows and takes a long-term perspective, fulfilling (R2) and (R3). Moreover, the determination of the terminal value considers the process' control flow. However, the example process includes only one exclusive choice and one simple merge (van der Aalst et al. 2003). How the terminal value could be calculated for more complex control flows is not explained. Hence, (R1) is only partly fulfilled. Although probabilities are included, thus considering risk to a certain extent, risk is not measured via the variance of the values, leaving (R4) unfulfilled. Overall, however, this work contributes significantly to the literature on value orientation in BPM.

Linderman et al. (2005) present a model for minimizing the expected costs of process maintenance. Although their approach considers costs and not cash flows, we regard (R2) as being partially fulfilled because this approach can be applied to cash flows as well. This work considers specific kinds of costs for a process as a whole, without considering the control flow; hence, (R1) is not fulfilled. As the authors do not determine the variance of the costs, risk is not considered, as is required in value-based BPM. Thus, (R4) is not fulfilled. A long-term perspective is included to some extent because average long-term costs are used. However, the time value of money is not incorporated. Therefore, (R3) is met in only a limited way.

Bai et al. (2007) and its most recent version, Bai et al. (2013), present a framework for determining where within a process to include control mechanisms for mitigating risk exposure. The paper focuses on the costs of executing a process to determine the best location. As with the previous paper, (R2) is partially fulfilled because the approach could have focused on cash flows instead. They consider risk measures such as expected loss, Value-at-Risk, and Conditional Value-at-Risk to determine the "optimal control structure design model." However, the variance is not included, leaving (R4) unfulfilled. Nevertheless, the paper contributes to the consideration of risks within BPM. The risk measures are determined with the help of simulations. Thus, the control flow is considered, fulfilling (R1). A long-term perspective is not included (R3), however.

The work of Buhl et al. (2011) also contributes to the value-oriented/value-based perspective in BPM. The rNPV is introduced as a process measure within value-based BPM, meeting the requirements of (R2) and (R3). Although the work argues that the variance of a process' NPV should be considered, methods of calculation are not discussed; thus, (R4) is not fulfilled.

Moreover, the paper remains on the corporate level rather than the operational process level, and control flow is thus not considered (R1).

Bolsinger et al. (2011) extend the work of Buhl et al. (2011) by providing detail about the rNPV, fulfilling (R2) and (R3). However, their paper also remains on the corporate level, without considering the operational process level, as required by (R1). Nor does the paper discuss how the variance can be determined (R4).

Sampath and Wirsing (2011) illustrate how the expected costs of a process can be determined using a process pattern based approach, which can also be applied to cash flows, partly fulfilling (R2). Since there is no consideration of costs in different periods, a long-term perspective is not included. This is also true for the calculation of the variance, which is not considered as well. Therefore, (R3) and (R4) are not fulfilled. Since the calculation of the costs is based on process patterns, the control flow of a process is considered. However, it is not stated, how to do so for a process that includes several different patterns. Nevertheless, (R1) is fulfilled to a considerable extent.

Wynn et al. (2013) incorporate the “cost perspective in the BPM Systems with the view to enable cost-aware process mining” (p. 87). This paper focuses on the reporting of costs, which could also be used for cash flows. As with previous papers, then, (R2) is partly fulfilled. The calculation of costs is confined to single process executions, without considering the long-term perspective, as required for (R3). A risk perspective is not incorporated; thus, (R4) is not fulfilled. The costs for all tasks within an execution are considered to determine the costs for a single process execution; process control flow is thus considered. However, the featured approach uses existing data about a process, which is possible only for existing processes and not for alternatives. Nevertheless, this approach fulfills (R1).

The contributions to the study of value-based BPM offered by the papers discussed above, all of which take a value-oriented/value-based perspective on BPM, are summarized in Table 1. Though the works all provide important contributions to value orientation in BPM, none fulfills every requirement. None of the works considers the operational process level, the long-term

Papers	<i>(R1) Control flow</i>	<i>(R2) Cash flow</i>	<i>(R3) Long-term perspective</i>	<i>(R4) Risk</i>
vom Brocke et al. (2010)	partly fulfilled	fulfilled	fulfilled	not fulfilled
Linderman et al. (2005)	not fulfilled	partly fulfilled	partly fulfilled	not fulfilled
Bai et al. (2007, 2013)	Fulfilled	partly fulfilled	not fulfilled	not fulfilled
Buhl et al. (2011)	not fulfilled	fulfilled	fulfilled	not fulfilled
Bolsinger et al. (2011)	not fulfilled	fulfilled	fulfilled	not fulfilled
Sampath and Wirsing (2011)	partly fulfilled	partly fulfilled	not fulfilled	not fulfilled
Wynn et al. (2013)	Fulfilled	partly fulfilled	not fulfilled	not fulfilled

Table 1 Summary of discussed papers with a value-oriented/value-based perspective

perspective, and risk together. Thus, none of the studies shows how to determine $E[NPV]$ and $Var[NPV]$ while considering processes' control flow, which is important to connect the corporate level with the operational process level (Rotaru et al. 2011; vom Brocke et al. 2010). Section 4 strives to close this gap by providing a valuation calculus for determining this expected value and process variance.

3 Illustrative Example

To provide a better understanding of the issues raised in the sections below, we briefly discuss an example of a process. We refer to this process whenever necessary to add an example in Section 4. In Section 5, we use the example process for evaluation purposes. Although the following valuation calculus is, of course, valid for more complex processes, we use this rather simple process, which nevertheless contains the five control flow patterns—XOR-split, XOR-join, AND-split, AND-join, and structured loop (van der Aalst et al. 2003)—for illustrative purposes.

Suppose there is an existing payroll process PR and a process alternative PR' , both of which are modified versions of real-world processes discussed in Neiger et al. (2006), as presented in Figure 2. The processes differ in their control flow, number of actions, and transition probabilities, which we briefly describe below. We use the term “action” for a fundamental component of a process, which “takes a set of inputs and converts them into a set of outputs” (Object Management Group 2011, p. 225), in line with the OMG Unified Modeling Language Superstructure (Object Management Group 2011).

The process PR has one action, “Enter Payroll run information” (a_1), with an expected cash outflow of \$1,000 per execution. This action is followed by two parallel actions, “Approve Payroll run” (a_2, a_3), each of which has an expected cash outflow of \$500 per execution. If data are entered incorrectly during the execution of the first action without being discovered and corrected in either of the following two actions, the expected cash outflow to fix the error

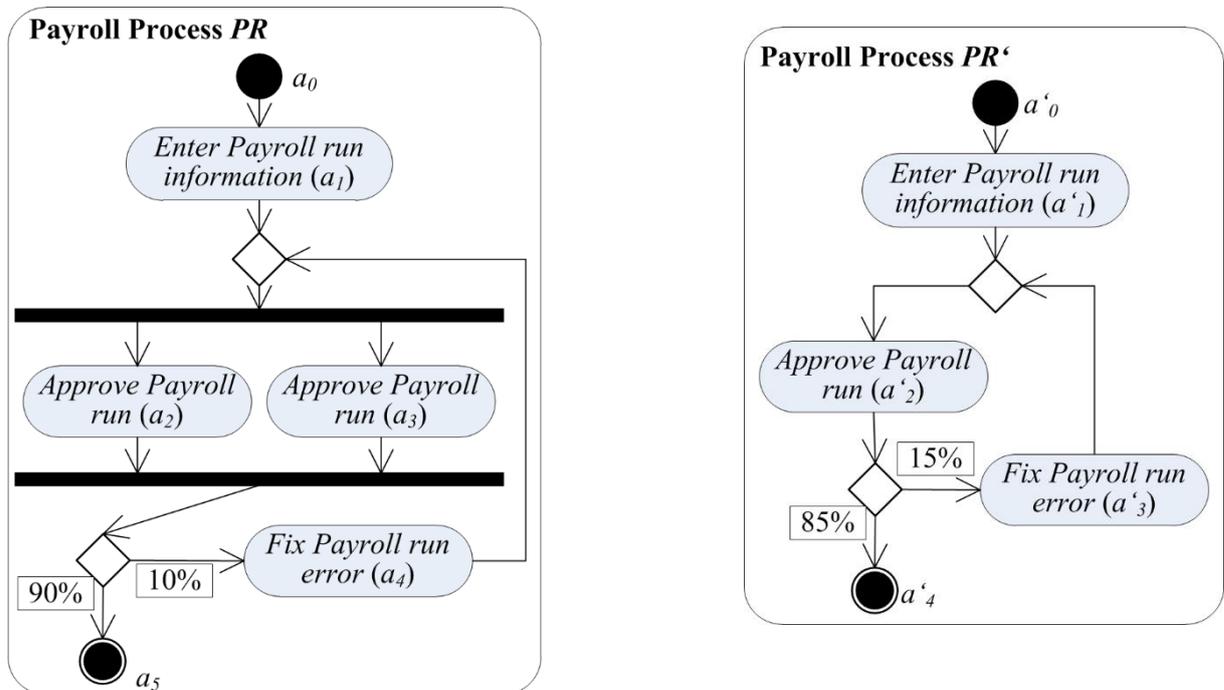


Figure 2 Existing payroll process PR and process alternative PR'

in the payroll run is \$5,000. This is done in the action “Fix Payroll run error” (a_4) and occurs with an estimated probability of 10%, which has to be approved again. Suppose that the process alternative PR' has only one action, “Approve Payroll run” (a'_2). The action “Fix Payroll run error” (a'_3) will then occur with an estimated probability of 15%, due to the less thorough approval.

The process manager’s challenge is to determine if the existing process PR is better or worse than PR' from a value-based BPM perspective. It is not easy just knowing the rNPV or the expected value, and particularly the variance of NPV . This is because the control flow structure of the processes needs to be considered. This structure can be very complex. Thus, the cash flows for the process’ actions need to be provided, and then the rNPV for the process as a whole can be calculated. If using a modeling tool that can calculate the rNPV, a process manager can determine if the existing process PR is better or worse than PR' in terms of the rNPV and how much better or worse it is.

4 Valuation Calculus

To determine the rNPV, as shown in expression (2), the expected value of the uncertain net present value of a process $E[NPV]$ and its variance $Var[NPV]$ need to be calculated. This is the focus of this section, whereas other papers deal with the determination of the risk aversion constant as the third component of the rNPV (see Section 2.1). Before we show how $E[NPV]$ and $Var[NPV]$ are connected with the process cash flow in Section 4.2, we state the assumptions of our valuation calculus in Section 4.1. Finally, in Section 4.3 we go into more detail about the process cash flow, while considering the control flow of a process.

4.1 Assumptions

The execution of a process is an important part of the determination of the expected value and variance. A closer look at the “execution of a process” and a more precise definition are necessary. Every time a process is executed, a *process instance* PI is performed. The Workflow Management Coalition (WfMC) defines a process instance in Hollingsworth and WfMC (2003) as the “representation of a single enactment of a process...including its associated data. Each instance represents a separate thread of execution...of the process...which may be controlled independently and will have its own internal state and externally visible identity” (p. 269). In order to specify an “enactment of a process” more precisely, we consider the term *process*. According to Hollingsworth and WfMC (2003), a process represents a “co-ordinated (parallel and/or serial) set of [actions] that are connected in order to achieve a common goal” (p. 275). When a process is executed (enacted) the whole set of actions is not necessarily executed, but only a subset, because there can be points in the “process where, based on a decision or workflow control data, one of several branches is chosen” (van der Aalst et al. 2003, p. 11). However, although the actions are connected to achieve a common goal, the process might fail to achieve the process goal because of errors in the process execution. Thus, a rather informal definition, similar to that in Braunwarth et al. (2010), is proposed below in order to ease the communication of the approach, which is in line with design science research.

Definition 1 (*Process instance and process path*). A *process instance* PI is the execution of a certain (sub)set of actions of a *process* (coordinated set of actions). The execution of this set is intended to achieve a common goal, has its own internal state, and an externally visible identity. In case of error, the set is only partly executed, and the process reaches the end of the process. Both a set of actions that achieves the process goal as well as the partly executed set form a path through the process, from start to end, called *process path* pp .

A process path is not necessarily a sequence of actions. It can include actions that are executed in parallel or executed more than once. Due to the structured loop in both processes seen in Figure 2, an infinite number of process paths is possible, although there is a finite number of actions, as, for example, in the left process in Figure 2, with one process path consisting of the actions a_1 , a_2 , and a_3 (case: no fixing is needed), another path of the actions a_1 , a_2 , a_3 , a_4 , a_2 , and a_3 (case: fixing is needed once), a third path with the actions a_1 , a_2 , a_3 , a_4 , a_2 , a_3 , a_4 , a_2 , and a_3 (case: fixing is needed twice), and so on. The number of different coordinated sets of actions is the number of process paths, which can be an infinite number, as is in the example in Figure 2. However, infinite numbers of process paths are uncommon. In reality, the probabilities at an exclusive choice would likely be very different every time a process instance reaches the same exclusive choice. In the example from Figure 2, with process PR it can be 90% and 10% the first time the exclusive choice is reached, 99% and 1% the second time, and 100% and 0% the third time. This eases the calculation because it results in a finite number of process paths while being closer to reality. This consideration about changing probabilities is possible with the expressions used below but is, to the best of our knowledge, not possible with any current process-modeling tool. A process instance executes exactly one possible process path. From this, we can make an assumption about how the considered processes are to be structured:

(A1) *A process P consists of a set A of actions $a_d \in A$, $d = 1, \dots, D$, one starting point a_0 , one final point a_{D+1} , transitions between the actions, and routing constructs (van der Aalst et al. 2003). A process instance PI starts in a_0 and ends in a_{D+1} . The probability that a process instance follows a process path pp_k is denoted by p_k , called “path probability.” Each path probability can be determined and is fixed. No logical error in the process can prevent a process instance from reaching a_{D+1} . The probability of an action’s execution failure is known.*

Within a process (model), identical tasks may be done more than once. For example, in the left process in Figure 2, “Approve Payroll run” is done twice, but we label one of them a_2 and the other a_3 . We consider everything modeled within a process as a different action, even if the same task is done, thus considering each to be a different action. This allows us to label all the tasks in a process differently in order to consider all of them separately in the valuation calculus. Action a_0 designates the (fictitious) point where the process starts, and a_{D+1} designates the (fictitious) point towards which a process instance proceeds and at which it always ends. The path probability p_k can be determined and is fixed (for more details on the determination of path probabilities, see appendix A). If no process action fails its execution, then every possible process instance starts in a_0 and ends in a_{D+1} . Hence, it is assumed that the process is correct and sound (van der Aalst et al. 2011). The execution of an action may fail with a known probability. Such failure of an action a_d can be modeled as an exclusive choice before a_d , with one choice going to a_{D+1} , which is taken with the probability that a_d fails, and a choice to continue the process, which is taken with the probability that a_d does not fail. Such explicit modeling of action failure would result in a new process path, to which a probability can be assigned. Thus, it is assumed that all known errors are modeled as described.

The cash flow of a process is caused by its actions. Thus, the cash flow of each action is important. Each action’s cash flow is caused by different action characteristics (e.g., wages, material). These characteristics result in different cash flows (e.g., cash outflow for wages, cash outflow for material; cp. vom Brocke et al. [2010]). In reality, the cash flow of an action might be different with each process instance. Hence, the cash flow of an action a_d is uncertain and

thus modeled as a random variable CF_{a_d} . In addition to the cash flows caused by actions, some cash flows are caused each time a process is executed, independent of the executed actions (e.g., cash outflows for overheads, cash inflows resulting from purchase transactions, cash outflows for process maintenance). These are cash flows of the characteristics of a whole process, called *process attributes*. These process attribute cash flows must be combined with the cash flows of actions to determine the cash flow of a process.

(A2) *The random variables CF_{a_d} represent the uncertain cash flows of the actions. The random variables $CF_{p_{a_s}}$, $s = 1, \dots, S$, represent the uncertain cash flows of process attributes, which are cash flows that are relevant for a process as a whole for every process instance. The expected values $E[CF_{a_d}]$ and $E[CF_{p_{a_s}}]$ as well as the variances $Var[CF_{a_d}]$ and $Var[CF_{p_{a_s}}]$ are finite and known.*

The expected value and variance of the cash flows of actions and of process attributes must be determined. Direct cash flows can be easily assigned to an action or process attribute. In terms of indirect cash outflows, Action-Based Costing can be used, as stated in Gulledge et al. (1997). This is also possible when accounting is linked with process-aware information systems (vom Brocke et al. 2011b). For cash inflows, the price of a product or service can be used and assigned to the process. Another possible method of determining the expected values and variances is to identify and use the subjective probability distributions of the cash flows. Suggestions on how to determine these distributions and elicit the necessary data from individuals can be found in Hubbard (2007).

Every planning horizon period contains several process instances, resulting in many process cash flows CF_p . Concerning the process instances, we assume the following:

(A3) *There are no dependencies between process instances.*

The process instances of a process are independent of the process instances of other processes; there is a high degree of autonomy (Feiler and Humphrey 1993). This is in line with Davamanirajan et al. (2006) because we concentrate on one process only. Moreover, process instances are independent of the process instances of the same process, as assumed in Bolsinger et al. (2011). In fact, a more general version of the valuation calculus is able to deal with dependencies through correlation coefficients. However, in order to prevent the presentation becoming overly complex, we assume independent process instances here.

4.2 Corporate Level

While the managers at the corporate level are interested in the rNPV, this value is based on the cash flows at the operational process level. Thus, the following expressions show how $E[NPV]$ and $Var[NPV]$ are connected with the process cash flow. With expression (1), it follows as expressed below:

$$E[NPV] = -I + \sum_{t=0}^T \frac{\sum_{j=1}^{n_t} E[CF_{P_j}]}{(1+i)^t} = -I + \sum_{t=0}^T \frac{n_t \cdot E[CF_{P_j}]}{(1+i)^t}. \quad (3)$$

It is $\sum_{j=1}^{n_t} E[CF_{P_j}] = n_t \cdot E[CF_{P_j}]$, because CF_{P_j} are identically distributed (Bolsinger et al. 2011). In combination with (A3), the random variables CF_{P_j} are independent and identically distributed (iid).

Then, it follows for $Var[NPV]$ that

$$Var[NPV] \stackrel{(A3)}{\cong} \sum_{t=0}^T \frac{\sum_{j=1}^{n_t} Var[CF_{P_j}]}{(1+i)^{2t}} \stackrel{iid}{\cong} \sum_{t=0}^T \frac{n_t \cdot Var[CF_{P_j}]}{(1+i)^{2t}}. \quad (4)$$

Hence, the corporate level puts the focus on the expected value of the process cash flow $E[CF_P]$ and its variance $Var[CF_P]$. In the following section, we show how $E[CF_P]$ and $Var[CF_P]$ are calculated including a consideration of the operational process level.

4.3 Operational Process Level

When a process instance “reaches” a routing construct upon which the process can “continue” in different ways (e.g., after an exclusive choice), the process instance “continues” depending on which condition(s) hold (e.g., depending on process inputs, on the environmental state). Thus, a process consists of multiple process paths, each executed with a certain probability. Every process path describes a possibility of executing a process from start to finish, which is why each process instance may result in a different cash flow depending on the control flow. This demonstrates the importance of process paths in considerations of processes as a whole. Thus, the expected value and variance of the cash flow of a single process path are first determined before the expected value and variance of the process as a whole are calculated.

Process Path

A process path pp_k contains actions from the start to the end of a process (see definition 1). Each process path is assigned a natural number k to make it formally distinct. The actions of a process path pp_k plus a_0 and a_{D+1} form (in a first step) an action multiset AS_k , whose elements are out of $A \cup \{a_0, a_{D+1}\}$. It is important that it be a multiset, so that loops can be considered, as the same actions can occur several times. Each action a_d in AS_k that occurs more than once (in a second step) is given an index $n \in \mathbb{N}$ in the form $a_d^{(1)}, a_d^{(2)}, \dots, a_d^{(n)}, \dots$. The index indicates the number of the loop iteration to which the action is assigned in order to distinguish among the actions, each of which is from different iterations, with different probabilities of being executed. In the process seen on the left in Figure 2, there are the action sets

$$AS_1 = \{a_0, a_1, a_2^{(1)}, a_3^{(1)}, a_5\},$$

$$AS_2 = \{a_0, a_1, a_2^{(1)}, a_3^{(1)}, a_4^{(1)}, a_2^{(2)}, a_3^{(2)}, a_5\},$$

$$AS_3 = \{a_0, a_1, a_2^{(1)}, a_3^{(1)}, a_4^{(1)}, a_2^{(2)}, a_3^{(2)}, a_4^{(2)}, a_2^{(3)}, a_3^{(3)}, a_5\}, \text{ and so on.}$$

The path probabilities are $p_1 = 0.9$, $p_2 = 0.1 \cdot 0.9 = 0.09$, $p_3 = 0.1^2 \cdot 0.9 = 0.009$ (for more details, see appendix A). Given that exactly one process path is taken if a process is executed and that they are mutually exclusive, the probabilities p_k sum up to 1. A process path has only sequential and parallel actions. Thus, the actions of a process path could be transformed into a sequential order without changing the result of the process path or the cash flow CF_{pp_k} of a process path pp_k . In addition to the cash flows of the actions, there are also the cash flows of process attributes CF_{pa_s} , which are considered with every execution of a process. Hence, it is

$$CF_{pp_k} = \sum_{a_d \in AS_k} CF_{a_d} + \sum_{s=1}^S CF_{pa_s}. \quad (5)$$

The expected value of CF_{ppk} is

$$E[CF_{ppk}] = \sum_{a_d \in AS_k} E[CF_{a_d}] + \sum_{s=1}^S E[CF_{pa_s}] \quad (6)$$

and the variance of CF_{ppk} is

$$\begin{aligned} & Var[CF_{ppk}] \\ &= \sum_{a_d \in AS_k} Var[CF_{a_d}] + \sum_{s=1}^S Var[CF_{pa_s}] + \sum_{\substack{a_d, a_j \in AS_k \\ d \neq j}} \rho_{a_d, a_j} \cdot \sigma_{a_d} \cdot \sigma_{a_j} \\ &+ 2 \sum_{a_d \in AS_k} \sum_{s=1}^S \rho_{a_d, pa_s} \cdot \sigma_{a_d} \cdot \sigma_{pa_s} + 2 \sum_{s=1}^{S-1} \sum_{j=s+1}^S \rho_{pa_s, pa_j} \cdot \sigma_{pa_s} \cdot \sigma_{pa_j}, \end{aligned} \quad (7)$$

where $Var[CF_{a_d}] = \sigma_{a_d}^2$ and $Var[CF_{pa_s}] = \sigma_{pa_s}^2$. The correlations ρ_{a_d, a_j} , ρ_{a_d, pa_s} , and ρ_{pa_s, pa_j} may reflect dependencies between the actions and process attributes. In Figure 2, it is possible that the lower the cash outflow of a_1 , (because the payroll run information is entered very quickly), the higher the cash outflow of a_2 and a_3 , (because they must make more corrections during the approval since the information was entered quickly and less carefully). Such dependencies could be reflected with correlations ρ_{a_1, a_2} and ρ_{a_1, a_3} . If there are no dependencies, all correlations ρ_{a_d, a_j} , ρ_{a_d, pa_s} , and ρ_{pa_s, pa_j} are zero, (7) simplifies to

$$Var[CF_{ppk}] = \sum_{a_d \in AS_k} Var[CF_{a_d}] + \sum_{s=1}^S Var[CF_{pa_s}]. \quad (8)$$

This determines the expected value and variance of the cash flow of one process path. The next step extends this to a process, where we need to consider all process paths at once in order to consider the control flow. Therefore, to determine the expected value and variance of a cash flow of a process, we must take into account the control flow of a process. A process may be not only a sequence of actions (as possible in a process path) but may also contain control flow patterns, like exclusive choice, simple merge, parallel split, synchronization, and loops (van der Aalst et al. 2003). Due to loops, each process can have an infinite number of process paths, which need to be considered using the valuation calculus below.

Process

To consider all process paths at once, a process is modeled as a probability space, which is a “triple $(\Omega, \mathcal{F}, \mathbf{P})$ of a sample space Ω , a [sigma]-algebra \mathcal{F} of sets in it, and a probability measure \mathbf{P} on \mathcal{F} ” (Feller 1971, p. 116).² This is a stochastic model that provides the formalism necessary for determining the expected value and variance of process cash flows. The sample space Ω is the set of all possibilities that the object under consideration can take; it is thus the set of all possible process paths. A sigma-algebra \mathcal{F} is a family of sets over Ω (a set of sets),

² The text is italicized in the source. The symbol \mathfrak{A} for the sigma-algebra and the symbol \mathfrak{S} for the text’s sample space were replaced by the now more commonly used symbols \mathcal{F} and Ω , respectively.

and a set in \mathcal{F} is called “event” (Feller 1971, p. 112). The probability measure \mathbf{P} assigns a certain probability to each event (Feller 1971, p. 115), thus to each set of process paths. In definition 2, a process is modeled as a probability space:

Definition 2 (*Process-probability-space*). A process P is a probability space $(\Omega, \mathcal{F}, \mathbf{P}_P)$ consisting of:

- the sample space $\Omega = \{pp_k \mid k \in \mathbb{N}\}$, which is the set of all possible process paths of a process P ,
- the sigma-algebra $\mathcal{F} = 2^\Omega$, which is the power set of Ω and therefore a set of subsets of Ω , which are the events of this probability space, and
- the probability measure

$$\mathbf{P}_P\{PP\} = \sum_{pp_k \in PP} f_{PI}(k) = \sum_{pp_k \in PP} p_k \quad \text{for all } PP \subseteq \Omega,$$

with the probability mass function

$$f_{PI}(k) = \text{Prob}(PI = k) = p_k,$$

where the process instance PI is a random variable

$$PI(\omega) = \begin{cases} 1 & \text{if } \omega = pp_1 \\ \dots & \dots \\ k & \text{if } \omega = pp_k \\ \dots & \dots \\ |\Omega| & \text{if } \omega = pp_{|\Omega|} \end{cases},$$

which takes on the value k for the k th process path with probability p_k .

In definition 2, a process is formally described as a probability space. In appendix B, it is formally shown that this process-probability-space is indeed a probability space. Definition 2 presents a process as a stochastic model and displays the formal differences and interplay among a process, a process instance, and a process path. As when modeling a process with UML activity diagrams, for example, a process model defines the process as a whole and does not change when a process is executed. The process paths are also fixed by the process model, which are fixed in the process-probability-space as well. As in every process, the process instance is the random component. Before executing a process, it is unknown which process path will be executed by a process instance; it could be any of them. In the process-probability-space, this randomness is represented by the random variable PI , which takes a certain process path pp_k with a certain probability p_k . Thus, in definition 2, it is possible to see a process, a process instance, and a process path explicitly within one model. If a process contains loops, an infinite number of process paths are possible. This is accounted for in definition 2 via the possibly infinite sample space. According to definition 2, the expected value of the cash flow of a process path pp_k is more precisely

$$E[CF_{pp_k}] = E[CF_P \mid PI = k]. \quad (9)$$

Expression (9) shows that the expected value of the cash flow of the process path pp_k is equal to the expected value of the cash flow of a process P given that process path pp_k is executed.

Now the expected value $E[CF_P]$ and the variance $\text{Var}[CF_P]$ of the cash flow of a process P can be determined. We want to express the expected value and variance only with the information about the actions and the additional process attributes.

In order to determine $E[CF_P]$ and $Var[CF_P]$, let $Pr(a_d)$ be the probability that an action $a_d \in AS$, with $AS := \bigcup_{k=1}^{|\Omega|} AS_k$, is executed when executing a process with

$$Pr(a_d) := \mathbf{P}_P\{PP_{a_d}\} = \sum_{pp_k \in PP_{a_d}} p_k \quad (10)$$

where PP_{a_d} is the set of process paths that contain the action a_d :

$$PP_{a_d} = \{pp_k \in \Omega \mid a_d \in AS_k\}. \quad (11)$$

It is $|\Omega|$ the number of process paths, which can be set to infinity for a process with loops. Expression (10), in combination with expression (11), shows that the probability that an action a_d is executed is the sum of the path probabilities p_k assigned to the process paths pp_k that contain action a_d . The expected value $E[CF_P]$ can be determined as follows, where (12) corresponds to the determination of expected costs in Linderman et al. (2005); for details, see appendix C:

$$E[CF_P] = \sum_{k=1}^{|\Omega|} E[CF_P \mid PI = k] \cdot Prob(PI = k) \quad (12)$$

$$= \sum_{k=1}^{|\Omega|} E[CF_{pp_k}] \cdot p_k \quad (13)$$

$$= \sum_{a_d \in AS} E[CF_{a_d}] \cdot Pr(a_d) + \sum_{s=1}^S E[CF_{pa_s}]. \quad (14)$$

The variance $Var[CF_P]$ can be similarly determined. Let $Pr(a_d, a_j)$ be the probability that both actions $a_d \in AS$ and $a_j \in AS$ are executed when executing a process with

$$Pr(a_d, a_j) := \mathbf{P}_P\{PP_{a_d, a_j}\} = \sum_{pp_k \in PP_{a_d, a_j}} p_k \quad (15)$$

where PP_{a_d, a_j} is the set of process paths that contain both actions a_d and a_j :

$$PP_{a_d, a_j} = \{pp_k \in \Omega \mid a_d \in AS_k, a_j \in AS_k, a_d \neq a_j\}. \quad (16)$$

Expression (15), in combination with expression (16), shows that the probability that both actions a_d and a_j are executed is the sum of the path probabilities p_k assigned to the process paths pp_k that contain both actions a_d and a_j .

The variance of the cash flow CF_P of a process P is (for details, see appendix D):

$$Var[CF_P] = \sum_{k=1}^{|\Omega|} E[(CF_P - E[CF_P])^2 | PI = k] \cdot Prob(PI = k) \quad (17)$$

$$= \sum_{k=1}^{|\Omega|} E[(CF_{pp_k} - E[CF_P])^2] \cdot p_k \quad (18)$$

$$= -E[CF_P]^2 + \sum_{k=1}^{|\Omega|} (Var[CF_{pp_k}] + E[CF_{pp_k}]^2) \cdot p_k \quad (19)$$

$$\begin{aligned} &= -E[CF_P]^2 + \sum_{a_d \in AS} (Var[CF_{a_d}] + E[CF_{a_d}]^2) \cdot Pr(a_d) \\ &+ \sum_{s=1}^S (Var[CF_{pa_s}] + E[CF_{pa_s}]^2) \\ &+ \sum_{a_d, a_j \in AS, a_d \neq a_j} (\rho_{a_d, a_j} \cdot \sigma_{a_d} \cdot \sigma_{a_j} + E[CF_{a_d}]E[CF_{a_j}]) \cdot Pr(a_d, a_j) \\ &+ 2 \sum_{a_d \in AS} \sum_{s=1}^S (\rho_{a_d, pa_s} \cdot \sigma_{a_d} \cdot \sigma_{pa_s} + E[CF_{a_d}]E[CF_{pa_s}]) \cdot Pr(a_d) \\ &+ 2 \sum_{s=1}^{S-1} \sum_{j=s+1}^S (\rho_{pa_s, pa_j} \cdot \sigma_{pa_s} \cdot \sigma_{pa_j} + E[CF_{pa_s}]E[CF_{pa_j}]). \end{aligned} \quad (20)$$

If there are no dependencies (i.e., if all correlations are zero) and no process attributes are considered—if, for example, it is the same for different process alternatives—(20) simplifies to

$$\begin{aligned} &Var[CF_P] \\ &= -E[CF_P]^2 + \sum_{a_d \in AS} (Var[CF_{a_d}] + E[CF_{a_d}]^2) \cdot Pr(a_d) \\ &+ \sum_{a_d, a_j \in AS, a_d \neq a_j} E[CF_{a_d}]E[CF_{a_j}] \cdot Pr(a_d, a_j). \end{aligned} \quad (21)$$

As expression (18) shows, the variance is the weighted average of the expected values of the squared difference between the cash flow of a certain process path and the expected value of the cash flow of the process. Although it might seem intuitive at first glance, it is not $CF_P = \sum_{k=1}^{|\Omega|} CF_{pp_k} \cdot p_k$.

Overall, with expression (14) and (20) in combination with expression (3) and (4), it is possible to determine $E[NPV]$ and $Var[NPV]$, which can then be used to calculate the rNPV with expression (2).

5 Evaluation

The evaluation of an artifact is an important step in design-oriented research, and various methods are available (Hevner et al. 2004; Peffers et al. 2008). Determining the utility of an artifact would be best achieved through a process-modeling tool that incorporates the valuation calculus and is used in a naturalistic setting with real users and real problems. However, this would be very time-consuming and resource-intensive. The evaluation framework for design science research presented in Venable et al. (2012) suggests performing the evaluation in an artificial setting. Sonnenberg and vom Brocke (2012) describe three evaluation activities (EVAL 1, EVAL 2, and EVAL 3) for such artificial settings. Each activity justifies a self-contained research contribution. We carry out all three activities to evaluate the artifact under study as follows:

EVAL 1: This activity is performed to justify the problem statement, research gap, and design objectives. This activity is conducted in sections 0 and 2.

EVAL 2: This activity validates the design specification and justifies the design tool/methodology. While Section 4 provides mathematical proofs and logical reasoning (formal deduction), valid evaluation methods for this activity, Section 5.1 shows the results of a feature comparison to illustrate the extent to which the stated design objectives of Section 2.2 are met. Section 5.2 “show[s] analytically that [the] artifact behaves as intended for a single test case” (Sonnenberg and vom Brocke 2012, p. 395) in order to demonstrate its feasibility. We therefore rely on the example introduced in Section 3.

EVAL 3: This activity validates an instance of the artifact in an artificial setting to prove its applicability. This is done in Section 5.3 by demonstrating how the artifact helped correct the calculation logic of the commercial process-modeling tool of the CubeFour company.

In addition to these three activities, in Section 5.4, we conduct a discussion regarding a competing artifact by comparing the valuation calculus with process simulations.

5.1 Feature Comparison

Section 2.2 outlines the four requirements (design objectives) for determining the rNPV. To verify if this paper contributes meaningfully to BPM research, we compare the valuation calculus with these requirements.

(R1) *Control flow*: The valuation calculus is based on path probabilities (see appendix A for details); it is thus based on the path that a process instance takes from the start to the end of a process. For each process that fulfills assumption (A1)—if the process is correct and sound and if its possible failures are known—all process paths can be determined. Process paths define how a process instance can reach the end of the process. Since process instances consider the control flow of a process and as process paths define the way of a process instance from start to finish, we consider the control flow of a process by using process paths for the valuation calculus. Although assumption (A1) is rather general, unknown failures (which exist when no process path considers them) are not considered in the valuation calculus. In any case, known or expected failures are considered.

(R2) *Cash flows*: The valuation calculus is designed to work for additive quantities, as shown by expression (5). Since the cash flows of the actions can be added to determine the rNPV, cash flows are considered in the described valuation calculus.

(R3) *Long-term perspective*: The calculation of the rNPV is based on the NPV presented in expression (1). The NPV considers the cash flows of future periods and the time value of money, incorporating a long-term perspective into value-based BPM.

(R4) *Risk*: To consider risk in value-based BPM, we must be able to measure it. Section 4.3 describes how the variance of *NPV* can be determined, which is used to measure risk.

Overall, while requirements (R2), (R3), and (R4) are fulfilled straightforwardly, some minor limitations regarding the control flow exist, as stated above (R1). However, assuming that we only consider correct and sound processes is feasible. Thus, we reduce the research gap considerably.

5.2 Illustrative Example (continued)

Let us again consider the payroll process *PR* introduced in Section 3 to demonstrate the feasibility of the valuation calculus. As illustrated in Section 4, determining the expected value and variance of the cash flow of a process is particularly challenging. We thus focus on this calculation. We first calculate the probability of each action (for detailed results see appendix E) based on expression (10). With expression (14), we then calculate the expected value:

$$\begin{aligned} E[CF_{PR}] &= E[CF_{a_1}] + E[CF_{a_2}] \cdot \sum_{i=0}^{\infty} 0.1^i + E[CF_{a_3}] \cdot \sum_{i=0}^{\infty} 0.1^i + E[CF_{a_4}] \cdot 0.1 \cdot \sum_{i=0}^{\infty} 0.1^i \\ &= 1,000 + 500 \cdot \frac{10}{9} + 500 \cdot \frac{10}{9} + 5,000 \cdot 0.1 \cdot \frac{10}{9} = 2,666.67. \end{aligned}$$

For the variance of CF_{PR} , we first calculate the probability $Pr(a_a, a_j)$ with expression (15) before determining the variance of CF_{PR} with expression (21); we do not consider any dependencies (for detailed results, see appendix F):

$$\begin{aligned} Var[CF_{PR}] &= -(E[CF_{PR}]^2) + (0 + 1,000^2) \cdot 1 + (0 + 500^2) \cdot \frac{10}{9} + (0 + 500^2) \cdot \frac{10}{9} + (0 + 5,000^2) \\ &\cdot 0.1 \cdot \frac{10}{9} + 2 \cdot \left[500 \cdot 1,000 \cdot \frac{10}{9} + 500 \cdot 500 \cdot \frac{10}{81} \right. \\ &+ 500 \cdot 1,000 \cdot \frac{10}{9} + 500 \cdot 500 \cdot \frac{110}{81} + 500 \cdot 500 \cdot \frac{10}{81} \\ &+ 5,000 \cdot 1,000 \cdot \frac{1}{9} + 5,000 \cdot 500 \cdot \frac{20}{81} + 5,000 \cdot 500 \cdot \frac{20}{81} + 5,000 \cdot 5,000 \cdot \frac{1}{81} \left. \right] \\ &= 2,108.19^2. \end{aligned}$$

In our example, there are no cash flows for the process as a whole CF_{pa_s} ($S = 0$). Thus, the sums in expression (20) that include CF_{pa_s} are zero. As a result, the payroll process *PR* has an expected cash outflow of 2,666.67, with a variance of 2,108.19². These numbers can also be calculated for the process alternative *PR'* in order to enable a comparison between process alternatives. The payroll process alternative *PR'* has an expected cash outflow of 2,470.59 with a variance of 2,506.05². In this case *PR* has a higher expected cash outflow than *PR'*, though the variance is lower, indicating a lower risk. We thus cannot decide if *PR'* improves *PR*. However, if we assume further parameters with expressions (3) and (4), we can calculate $E[NPV]$ and $Var[NPV]$. In a last step, we can incorporate these values with expression (2), which results in one value for *PR* and one value for *PR'* for comparison.

Although, as mentioned, there is likely not an infinite number of process paths, this example shows that it is possible to consider such a case.

5.3 The Case of CubeFour

The following is a case presentation describing how the insights in Section 4 helped correct the calculation capabilities of the “cube4process” process-modeling tool used by CubeFour. Although the capabilities of cube4process were already more advanced than those of most other tools, we were able to help improve these capabilities using our valuation calculus.

Cube4process enables its users to not only model processes but also add financial information, such as the (expected) cash flow of an action’s execution. This information can be added to every action. The probabilities of each transition within the process model can be added as well, which can then be used to determine the path probabilities (see appendix A for details). With this information, the tool provides the expected cash flow of the process analytically. The tool supports the basic control flow patterns XOR-split, XOR-join, AND-split, and AND-join (van der Aalst et al. 2003) as well as loops (with some minor exceptions). The tool is also intended to support OR-splits. However, after reviewing the tool based on the mathematical insights in this paper, it was discovered that OR-splits, in particular, add extra complexity to the determination of the expected cash flow, as described below.

Consider the process seen in Figure 3. After an OR-split, the process continues with, depending on the transition conditions, only one transition, any combination of two transitions, or even all three transitions. The transitions are not mutually exclusive, as with a XOR-split. This is why the transition probabilities in Figure 3 do not add up to 1. Thus, in 60% of the process instances, action *B* is executed after action *A*. Action *C* is executed after *A* in 50% of the process instances, and *D* after *A* in 10% of the process instances.

Using cube4process, the process in Figure 3 is modeled as presented in Figure 4 (Task 1:= action *A*, Task 2:= action *B*; Task 3:= action *C*, Task 4:= action *D*, and Task 5:= action *E*). Below each action, one can see the additional information regarding the cash flows of the

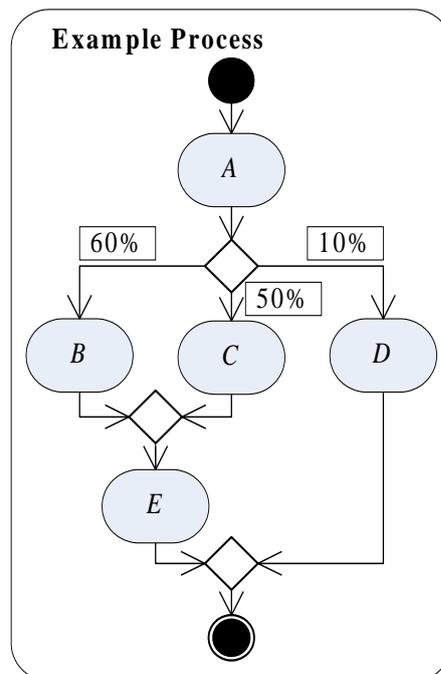


Figure 3 Example process with OR-split

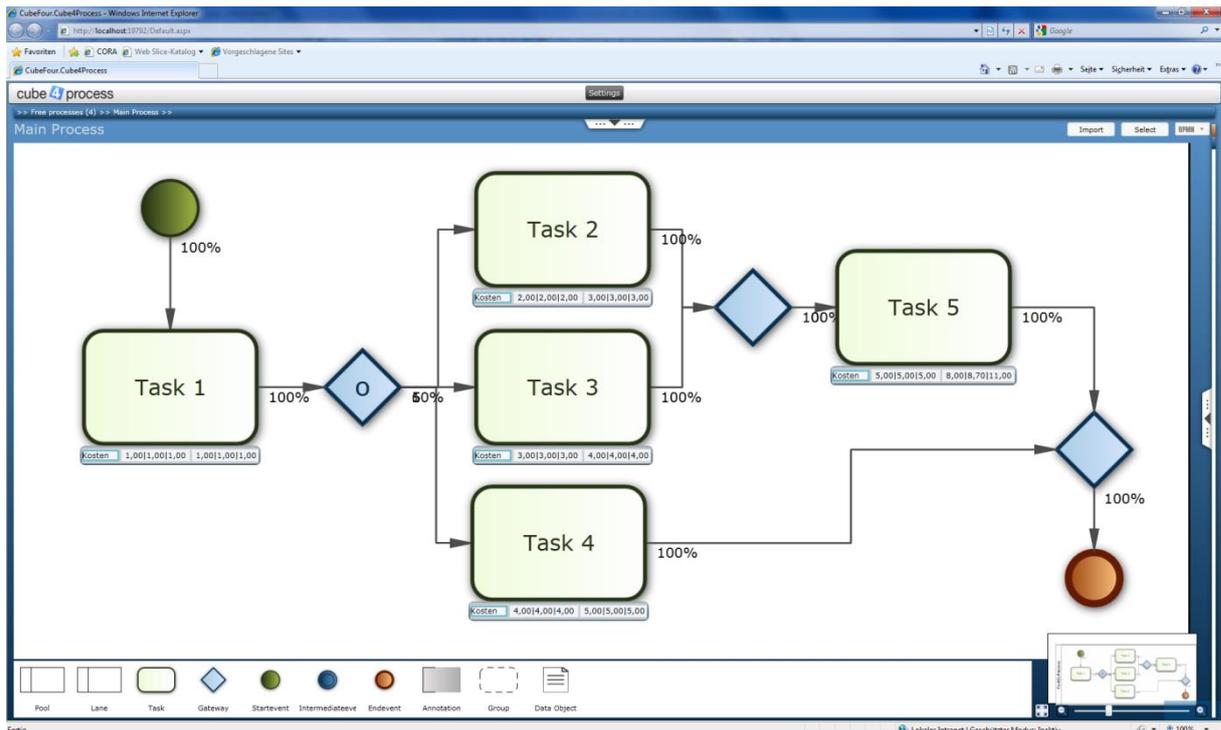


Figure 4 The process in Figure 3 modeled with cube4process

action's execution. The first number gives the minimal cash flow of an execution, the second number is the average cash flow, and the third number is the maximal cash flow. The fourth number is the minimal cash flow of the whole process from the start until after the execution of the action. The fifth number is the corresponding average cash flow, and the sixth number is the maximal cash flow.

The information regarding the transition probabilities is important for reaching the correct determination of the expected value because this will determine the probability that an action will be executed when the process is executed. It is easy to see that $Pr(A) = 1$, $Pr(B) = 0.6$, $Pr(C) = 0.5$, and $Pr(D) = 0.1$. However, what is the probability that action E will be executed? Figure 5 provides an overview of the determination of cube4process about the probabilities and the expected value before the correction through the mathematical insights by this paper. The probability that E will be executed is given as 0.8 and the expected value as 8.1. CubeFour used the addition law of probability for this calculation. The tool made the following calculation:

$$Pr(E) = Pr(B) + Pr(C) - Pr(B) \cdot Pr(C) = 0.6 + 0.5 - 0.6 \cdot 0.5 = 0.8.$$

Here, it is implicit that each action is an event; thus, the calculation is based on a probability space whose sample space Ω is the set of all the actions of a process. However, it can be shown that a process cannot be modeled as a probability space based on actions as the events. Hence, as the calculation is not based on a valid probability space, it cannot be guaranteed to provide correct results. This holds true for all control flows and can best be illustrated by a process that contains an OR-split, which is why this construct is the focus of this section.

After considering the valuation calculus of this paper, all calculations, if implemented correctly, will lead to valid results because, in definition 2, this paper provides a valid probability space that provides the foundation for a correct calculation of the probability. Here, the probability that an action will be executed is given by expression (10) with

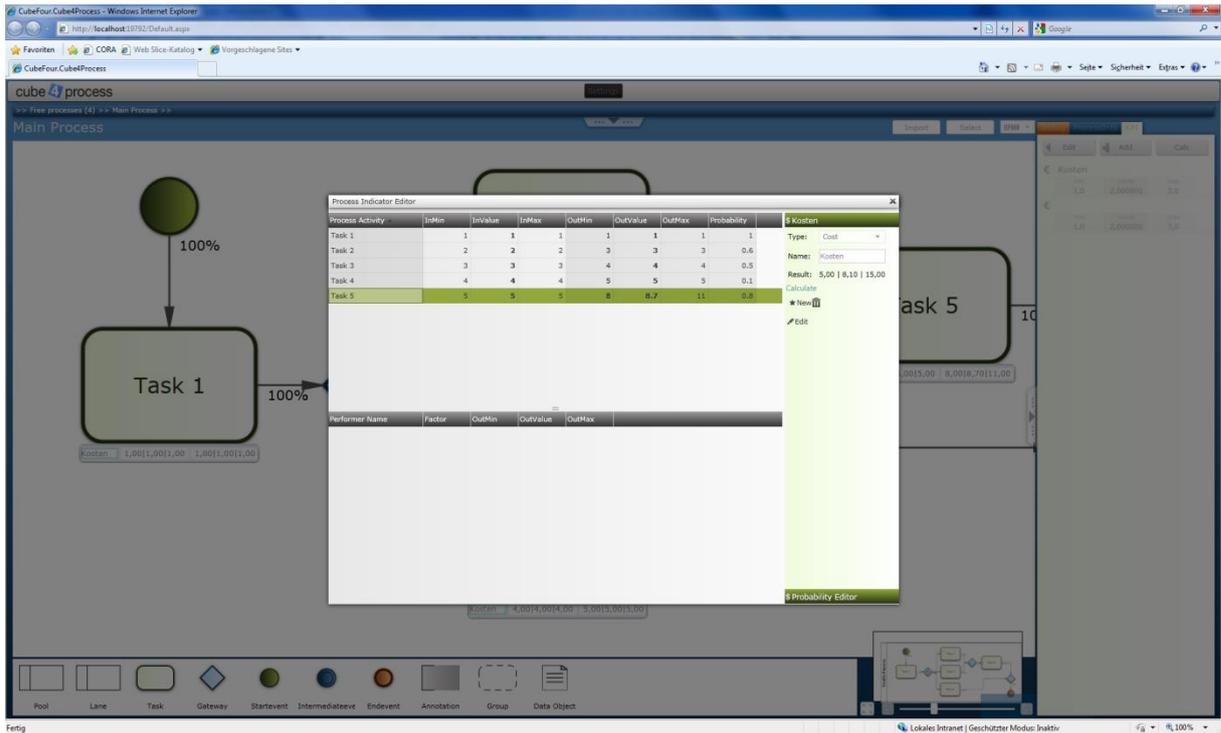


Figure 5 Results of the analytical determination of the probabilities and the expected value

$$Pr(a_d) = \sum_{pp_k \in PP_{a_d}} p_k .$$

The probability that action a_d will be executed is the sum of the path probabilities of the process paths in which the action takes part. However, as we briefly illustrate below, it is impossible to calculate the probability that action E will be executed with the given information using this valid method. The given transition probabilities 60%, 50%, and 10% do not give enough information to enable a determination of the path probabilities and thus the probability that an action will be executed. This is because, for example, the 60% indicates only that action B is executed in 60% of the process instances, but does not indicate in how many of these process instances action C or D is also executed, information necessary for determining the probability of each process path. The problem is illustrated by the two examples of path probabilities in Table 2 and Table 3. First, let us assume that the path probabilities are given according to the values in Table 2.

process path	pp_1	pp_2	pp_3	pp_4	pp_5	pp_6	pp_7
actions	A, B, E	A, C, E	A, D	A, B, C, E	A, B, D, E	A, C, D, E	A, B, C, D, E
path probability p_k	0.43	0.37	0.02	0.1	0.05	0.01	0.02

Table 2 Actions and path probabilities of all process paths

Then it is:

$$Pr(B) = p_1 + p_4 + p_5 + p_7 = 0.43 + 0.1 + 0.05 + 0.02 = 0.6,$$

$$Pr(C) = p_2 + p_4 + p_6 + p_7 = 0.37 + 0.1 + 0.01 + 0.02 = 0.5,$$

$$Pr(D) = p_3 + p_5 + p_6 + p_7 = 0.02 + 0.05 + 0.01 + 0.02 = 0.1,$$

$$Pr(E) = p_1 + p_2 + p_4 + p_5 + p_6 + p_7 = 0.43 + 0.37 + 0.1 + 0.05 + 0.01 + 0.02 = 0.98.$$

Let us assume that the path probabilities would be slightly different according to the values in Table 3.

process path	pp_1	pp_2	pp_3	pp_4	pp_5	pp_6	pp_7
actions	A,B,E	A,C,E	A,D	A,B,C,E	A,B,D,E	A,C,D,E	A,B,C,D,E
path probability p_k	0.43	0.36	0.03	0.11	0.04	0.01	0.02

Table 3 Actions and slightly changed path probabilities of all process paths

Then it still is

$$Pr(B) = p_1 + p_4 + p_5 + p_7 = 0.43 + 0.11 + 0.04 + 0.02 = 0.6,$$

$$Pr(C) = p_2 + p_4 + p_6 + p_7 = 0.36 + 0.11 + 0.01 + 0.02 = 0.5,$$

$$Pr(D) = p_3 + p_5 + p_6 + p_7 = 0.03 + 0.04 + 0.01 + 0.02 = 0.1.$$

However, it is

$$Pr(E) = p_1 + p_2 + p_4 + p_5 + p_6 + p_7 = 0.43 + 0.36 + 0.11 + 0.04 + 0.01 + 0.02 = 0.97.$$

Thus, although we do not change the information provided in Figure 3 because the probabilities of action B, C, and D do not change, the probability of action E changes, which also changes the expected value of the process. Therefore, the transition probabilities seem insufficient for considering OR-splits during the calculation of the expected value. Additional information about the probability for the combination of the actions after an OR-split is required.

The developers of cube4process were given an insight into the mathematical foundation of processes. As a result, CubeFour was able to correct the calculation of their tool, providing a mathematically sound calculation of the expected value and creating a valuable asset for use in process improvement projects.

5.4 Comparison with Process Simulations

Section 4 describes the focus placed on the expected value of a process cash flow and its variance because these are central to the determination of the expected value and variance of a process' NPV. In Section 4.3, we show how they can be calculated. However, they could also be determined via process simulations, which are thus a competing artifact. In Table 4, we therefore compare our valuation calculus with process simulations to determine the expected value of a process cash flow and its variance using the criteria we consider most distinctive.

	Process simulation (PS)	Valuation Calculus
Expressiveness	A PS can explicitly consider various factors such as time, costs, and resource restrictions. However, if the PS aims to determine a monetary value for a process, then the question arises how factors like resource restrictions are transformed into monetary values.	The presented valuation calculus takes on a value-oriented/value-based perspective. Thus, factors like time and resource restrictions have to be transformed into cash flows to be considered. While this might be possible with some factors, it is challenging with others.
Process complexity	A PS is able to handle processes with a very complex control flow. However, increasing complexity increases the runtime of a PS.	When implemented by a tool, the determination of the rNPV may be impossible for processes with a very complex control flow, though theoretically possible according to our valuation calculus, or the runtime for the calculation may be very high, even higher than with a PS.
Information needed	The structure of the process, the transition probabilities, and the probability distribution of CF_{ad} and CF_{pas} .	The structure of the process, the transition probabilities, and only the expected value and the variance of CF_{ad} and CF_{pas} .
Precision of results	A PS delivers imprecise results (Sun et al. 2006), which means that the calculation cannot be repeated in a manner that leads to the same result with every run (Pearn et al. 1998). It is a technique that can approximate the expected value and variance, but it cannot provide the correct value (van Hee and Reijers 2000). However, the more extensive the PS, the higher its precision.	The valuation calculus provides precise results.
Sensitivity analysis	A PS supports “what-if” analysis (van der Aalst 2001) to determine how the result of a process changes if, for example, one factor is changed at a time. Because of its lack of precision, however, the extent to which a changed result is due solely to the changed factor cannot be precisely determined. A change in a result could be due to the imprecision of the PS.	If a process is modified, the rNPV can be calculated again, which allows for the determination of whether a process improved from a value-based perspective to the process change. Thus, a “what-if” analysis is possible with the valuation calculus as well. This analysis is precise and thus indicates if the change in the result is due solely to the change of the process.

Table 4 Comparison of process simulation and the analytical approach of this paper

Overall, we consider process simulations to be advantageous in their expressiveness and their treatment of processes with complex control flows. However, we consider this paper's approach to be advantageous in terms of required information and its precision in determining the expected value and variance. Particularly beneficial is the fact that, because we do not need to know the whole probability distribution of CF_{ad} and CF_{pas} , the presented valuation calculus might encourage a broader use in practice.

6 Conclusion and Outlook

Process measures are important instruments for analyzing processes and deciding on process changes. For the decision making-oriented branch of value-based BPM, the rNPV of a process is an important process measure. However, current research on value-based BPM provides the rNPV on only the corporate level. Thus, this paper connects the corporate level with the operational process level, providing a valuation calculus that considers the control flow of processes. This paper contributes to value-based BPM in the following ways:

1. This paper develops its valuation calculus such that the rNPV of a process can be calculated, bringing value-based BPM to the operational process level and allowing it to be implemented via process-modeling tools. A modeling tool with such calculation capabilities is a valuable asset to any process manager who needs to decide among various process alternatives while considering the principles of value-based management.
2. The paper provides a theoretical foundation for more formal research in BPM, while already making a valuable contribution to practice, as seen in Section 5.3. The valuation calculus has helped improve the calculation capabilities of a commercial process-modeling tool currently being developed by CubeFour.
3. Finally, since the paper's formalism in calculating the expected value and variance is based on the fact that the cash flow of a process path is the sum of the cash flows of the actions in that path, this formalism is usable not only for cash flows but for any kind of additive quantity, such as costs, energy, or used material.

Despite the contributions of this paper to BPM research and practice, it has limitations that point to possibilities for future study:

1. The first limitation regarding the calculation of the rNPV is assumption (A3), that there are no dependencies among process instances, as made in other works (Bolsinger et al. 2011; Davamanirajan et al. 2006). A more general version of the valuation calculus could consider these dependencies via correlation coefficients. However, this version would make the presentation of the valuation calculus overly complex. The presented simplification eases the communication of the valuation calculus significantly.
2. Another limitation regarding the applicability of the valuation calculus is the availability of necessary data. Along with the need to determine the expected value of the action cash flow and its variance is the need to determine the path probabilities. Doing so requires information regarding the transition probabilities from action to action. The transition probabilities could be estimated by an expert (Hubbard 2007) or by analyzing process log files (zur Muehlen and Shapiro 2010) using, for example, a process-mining framework like ProM (Rubin et al. 2007). Furthermore, tapping the full potential of the variance requires that the correlations be determined. Gathering these data is possible, particularly when process-mining techniques are used, but it is not easy.

3. Value-based BPM is based on monetary values and uses cash flows as the common denominator. This common denominator allows a comparison among various process alternatives. However, different performance dimensions are typically used in BPM, such as time, cost, quality, and flexibility (Reijers and Liman Mansar 2005). While costs are already a monetary value, the other dimensions need to be monetized for the presented valuation calculus. Of the other dimensions, time, in combination with wages, can most readily be transformed into monetary values. While quality and flexibility are more challenging, some papers focus on the transformation of flexibility into monetary values (Braunwarth and Ullrich 2010; Neuhuber et al. 2013). Thus, although having a common denominator is an advantage, much more research on converting other BPM goals/non-monetary dimensions into monetary values is required. It will then be possible for value-based BPM to exploit its full potential as a comprehensive framework for BPM decisions by supporting the improvement of processes through a monetary-centered view of BPM.
4. As discussed in Section 5.4, process simulations can probably be used more conveniently with more processes than can an implemented version of the valuation calculus, as processes can be very complex. For complex processes with a high number of process paths, the expected value and variance must be calculated automatically because manual calculation would be very time-consuming. Algorithms are thus needed to determine the path sets and path probabilities. These have not been sufficiently explored. Some algorithms can calculate path sets (Byers and Waterman 1984) using depth-first search. However, as these algorithms are not specifically for processes, they do not consider all control flow patterns nor calculate path probabilities. The depth-first search is widely used and well-studied (Sedgewick and Schidlowsky 2003). Thus, a depth-first search algorithm can be used to get all path sets and calculate the path probabilities while considering the control flow patterns. However, the runtime of such algorithms could be high for complex processes.
5. Finally, since processes can be very complex, a more formal and extensive assessment than that given in Section 5.1 is needed to determine the extent of the valuation calculus' validity for different kinds of processes. Processes are complex not only from a control flow perspective but also from, for example, resource, data, time, and function perspectives (van der Aalst 2013). Such different perspectives need to be subject to further research.

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Appendix

A. Determination of Path Probabilities

To determine the expected value of a process cash flow and its variance, it is essential to determine the path probabilities p_k . This is presented in the following. During a process improvement project, a process is presented as a process model with a process-modeling tool. With the help of this formal presentation, it is possible to formally describe, how a path probability p_k is determined. In order to do so, the process model (as defined in Hollingsworth and WfMC [2003, p. 266]) of a process P is defined as a graph G .

The process model of a process P is a graph, because a process model is a set of nodes (*vertices*) that are interconnected by arrows (*edges*) (Gibbons 1985). The set of vertices is denoted by V and the set of edges by E and we write $G = (V, E)$. Because the edges are arrows, a process is a directed graph (Gibbons 1985). More precisely, we assume that a process model of a process P is defined as a graph G as followed:

- (D1) A process model of a process P is a directed graph $G = (V, E)$ with one root vertex a_0 and one final vertex a_{D+1} , toward which all edges are directed. It is V the set of vertices and E the set of edges.
- (D2) The set V consists of the set of actions A united with the set RC of the routing constructs (van der Aalst et al. 2003) to denote control flow patterns of P , a_0 and a_{D+1} , i.e., $V := A \cup RC \cup a_0 \cup a_{D+1}$.
- (D3) A contains all D actions of P , numbered from 1 to D .
- (D4) RC is the set of the routing constructs to denote the control flow patterns, e.g., XOR-split, XOR-join, AND-split and AND-join (van der Aalst et al. 2003). Each element has one distinct index. For example, in Figure 2 (left process) these vertices are *XOR-join₁*, *AND-split₂*, *AND-join₃* and *XOR-split₄*.
- (D5) The edge-set E contains all the directed edges between the vertices. The directed edge (v_i, v_j, p_{ij}) is a member of the set E if and only if there is an arrow between vertex $v_i \in V$ and $v_j \in V$, pointing from v_i to v_j and having a probability for this transition (Hollingsworth and WfMC 2003, p. 282) of p_{ij} , with $0 < p_{ij} \leq 1$, as weight. Each vertex in A has exactly one edge pointing toward it and exactly one edge pointing away from it.

The actions and routing constructs of a process path pp_k plus a_0 and a_{D+1} form (in a first step) a path multiset PS_k , whose elements are out of V . The fact that it is a multiset is important to consider loops, as then the same vertices of G can occur several times. Each vertex v_i in PS_k that occurs more than once (in a second step) is given an index $n \in \mathbb{N}$ in the form $v_i^{(1)}, v_i^{(2)}, \dots, v_i^{(n)}, \dots$. The index indicates the number of the iteration of a loop that the vertex is assigned to. This is to distinguish the vertices from one another because each of them is from different iterations that have different probabilities of being executed. In the left process in Figure 2, there are the path sets

$$PS_1 = \{a_0, a_1, XOR-join_1^{(1)}, AND-split_2^{(1)}, a_2^{(1)}, a_3^{(1)}, AND-join_3^{(1)}, XOR-split_4^{(1)}, a_5\},$$

$$PS_2 = \{a_0, a_1, XOR-join_1^{(1)}, AND-split_2^{(1)}, a_2^{(1)}, a_3^{(1)}, AND-join_3^{(1)}, XOR-split_4^{(1)}, a_4^{(1)}, XOR-join_1^{(2)}, AND-split_2^{(2)}, a_2^{(2)}, a_3^{(2)}, AND-join_3^{(2)}, XOR-split_4^{(2)}, a_5\}$$

and so on, with $v_1 := a_0$, $v_2 := a_1$, $v_3^{(1)} := XOR-join_1^{(1)}$, $v_3^{(2)} := XOR-join_1^{(2)}$, \dots , $v_4^{(1)} := AND-split_2^{(1)}$, $v_4^{(2)} := AND-split_2^{(2)}$, \dots , $v_5^{(1)} := a_2^{(1)}$, $v_5^{(2)} := a_2^{(2)}$, \dots , $v_6^{(1)} := a_3^{(1)}$, $v_6^{(2)} := a_3^{(2)}$, \dots , $v_7^{(1)} := AND-$

$join_3^{(1)}$, $v_7^{(2)} := AND-join_3^{(2)}$, ..., $v_8^{(1)} := XOR-split_4^{(1)}$, $v_8^{(2)} := XOR-split_4^{(2)}$, ..., $v_9^{(1)} := a_4^{(1)}$, ..., and $v_{10} := a_5$.

Every process path pp_k is executed with a certain *path probability* p_k that is the product of the transition probabilities of process path pp_k :

$$p_k = \prod_{v_i^{(m)}, v_j^{(n)} \in PS_k} p_{i^{(m)}j^{(n)}} \quad \text{for all } p_{i^{(m)}j^{(n)}} > 0. \quad (22)$$

The transition probability $p_{i^{(m)}j^{(n)}}$ that $v_i^{(m)}$ is followed by $v_j^{(n)}$ can be estimated and is fixed. These transition probabilities could be estimated by an expert (Hubbard 2007) or by analyzing process log files (zur Muehlen and Shapiro 2010) using, for example, a process-mining framework like ProM (Rubin et al. 2007). In the left process in Figure 2, for example, for the process path pp_1 there are the (non-zero) transition probabilities $p_{12} = 1$, $p_{23^{(1)}} = 1$, $p_{3^{(1)}4^{(1)}} = 1$, $p_{4^{(1)}5^{(1)}} = 1$, $p_{4^{(1)}6^{(1)}} = 1$, $p_{5^{(1)}7^{(1)}} = 1$, $p_{6^{(1)}7^{(1)}} = 1$, $p_{7^{(1)}8^{(1)}} = 1$ and $p_{8^{(1)}10} = 0.9$. All other transition probabilities are zero. Then it is

$$\begin{aligned} p_1 &= \prod_{v_i^{(m)}, v_j^{(n)} \in PS_1} p_{i^{(m)}j^{(n)}} \\ &= \underbrace{1}_{a_0 \text{ to } a_1} \cdot \underbrace{1}_{a_1 \text{ to } XOR-join_1} \cdot \underbrace{1}_{XOR-join_1 \text{ to } AND-split_2} \cdot \underbrace{1}_{AND-split_2 \text{ to } a_2} \cdot \underbrace{1}_{AND-split_2 \text{ to } a_3} \\ &\cdot \underbrace{1}_{a_2 \text{ to } AND-join_3} \cdot \underbrace{1}_{a_3 \text{ to } AND-join_3} \cdot \underbrace{1}_{AND-join_3 \text{ to } XOR-split_4} \cdot \underbrace{0.9}_{XOR-split_4 \text{ to } a_5} = 0.9 \end{aligned}$$

and

$$p_2 = \prod_{v_i^{(m)}, v_j^{(n)} \in PS_2} p_{i^{(m)}j^{(n)}} = 0.09, \text{ etc.}$$

Expression (22) is not true in the event that a process model contains an OR-split (van der Aalst et al. 2003). This fact is important in Section 5.3, when showing how this valuation calculus helped to improve the calculation capabilities of a process-modeling tool. However, every OR-split can formally be transformed into a composition of XOR-splits and AND-splits, which allows the use of expression (22). Otherwise, the path probabilities need to be estimated.

B. Process-Probability-Space

In probability theory, “a probability space is a triple (Ω, \mathcal{F}, P) of a sample space Ω , a [sigma]-algebra \mathcal{F} and a probability measure P on \mathcal{F} ” (Feller 1971, p. 116). The sample space Ω is the set of all possibilities that the object under consideration can take; it is thus the set of all possible process paths, as these represent all possibilities of a process execution. A sigma-algebra has properties such that:

- (i) “If a set A is in \mathcal{F} so is its complement $[A^C = \Omega \setminus A]$.”
- (ii) “If $\{A_n\}$ is any countable collection of sets in \mathcal{F} , then also their union $\cup A_n$ and intersection $\cap A_n$ belong to \mathcal{F} ” (Feller 1971, p. 112).

That the sigma-algebra in definition 2 is the power set of the set of all process paths means that (i) and (ii) are fulfilled.

“A probability measure \mathbf{P} on a [sigma]-algebra \mathcal{F} of sets in Ω is a function assigning a value $\mathbf{P}\{A\} \geq 0$ to each set A in \mathcal{F} such that $\mathbf{P}\{\Omega\} = 1$ and that for every countable collection of non-overlapping sets A_n in \mathcal{F} [it is] $\mathbf{P}\{\cup A_n\} = \sum_n \mathbf{P}\{A_n\}$ ” (Feller 1971, p. 115).

All process paths are mutually exclusive, and they represent all possibilities how a process can be executed. Every process path pp_k is executed with a certain *path probability* $p_k > 0$. Given that there is exactly one process path taken if a process is executed and that they are mutual exclusive, the probabilities p_k sum up to 1, fulfilling $\mathbf{P}\{\Omega\} = 1$. The property $\mathbf{P}\{\cup A_n\} = \sum_n \mathbf{P}\{A_n\}$ also holds for every countable collection of non-overlapping sets A_n in \mathcal{F} since \mathcal{F} is the power set of Ω .

C. Expected Value of the Process Cash Flow

Let the probability that an action $a_d \in AS$, with $AS := \cup_{k=1}^{|\Omega|} AS_k$, is executed when executing a process be

$$Pr(a_d) := \mathbf{P}_P\{PP_{a_d}\} = \sum_{k=1}^{|\Omega|} p_k \cdot \mathbb{I}_{AS_k}(a_d)$$

with the indicator function

$$\mathbb{I}_{AS_k}(a_d) = \begin{cases} 1, & a_d \in AS_k \\ 0, & a_d \notin AS_k \end{cases}$$

and the set PP_{a_d} of process paths in which the action a_d is

$$PP_{a_d} = \{pp_k \in \Omega | a_d \in AS_k\}.$$

Then it is:

$$\begin{aligned} E[CF_P] &= \sum_{k=1}^{|\Omega|} E[CF_P | PI = k] \cdot Prob(PI = k) = \sum_{k=1}^{|\Omega|} E[CF_{pp_k}] \cdot p_k \\ &= \sum_{k=1}^{|\Omega|} \left(p_k \cdot E \left[\sum_{a_d \in AS_k} CF_{a_d} + \sum_{s=1}^S CF_{pa_s} \right] \right) = \sum_{k=1}^{|\Omega|} \left(p_k \cdot \left(\sum_{a_d \in AS_k} E[CF_{a_d}] + \sum_{s=1}^S E[CF_{pa_s}] \right) \right) \\ &= \sum_{k=1}^{|\Omega|} \left(\sum_{a_d \in AS_k} p_k \cdot E[CF_{a_d}] \right) + \sum_{k=1}^{|\Omega|} \left(p_k \cdot \sum_{s=1}^S E[CF_{pa_s}] \right) \\ &= \sum_{k=1}^{|\Omega|} \left(\sum_{a_d \in AS_k} \mathbb{I}_{AS_k}(a_d) \cdot p_k \cdot E[CF_{a_d}] \right) + \sum_{s=1}^S E[CF_{pa_s}] \cdot \underbrace{\sum_{k=1}^{|\Omega|} p_k}_{=1} \\ &= \sum_{k=1}^{|\Omega|} \left(\sum_{a_d \in AS} \mathbb{I}_{AS_k}(a_d) \cdot p_k \cdot E[CF_{a_d}] \right) + \sum_{s=1}^S E[CF_{pa_s}] \end{aligned}$$

$$\begin{aligned}
&= \sum_{a_d \in AS} \left(\sum_{k=1}^{|\Omega|} \mathbb{I}_{AS_k}(a_d) \cdot p_k \cdot E[CF_{a_d}] \right) + \sum_{s=1}^S E[CF_{pa_s}] \\
&= \sum_{a_d \in AS} E[CF_{a_d}] \left(\sum_{k=1}^{|\Omega|} p_k \cdot \mathbb{I}_{AS_k}(a_d) \right) + \sum_{s=1}^S E[CF_{pa_s}] \\
&= \sum_{a_d \in AS} E[CF_{a_d}] \cdot Pr(a_d) + \sum_{s=1}^S E[CF_{pa_s}]
\end{aligned}$$

D. Variance of the Process Cash Flow

In the following first step, it is shown that $Var[CF_P] = -E[CF_P]^2 + \sum_{k=1}^{|\Omega|} p_k \cdot E[CF_{pp_k}^2]$ in two ways. The first way is similar to the beginning of the calculation for the expected value in appendix C. The second way is more detailed and includes $\sum_{k=1}^{|\Omega|} E[(CF_{pp_k} - E[CF_P])^2] \cdot p_k$, a more intuitive expression for $Var[CF_P]$. This is why both ways are presented.

Way 1

$$\begin{aligned}
Var[CF_P] &= E[CF_P^2] - E[CF_P]^2 = -E[CF_P]^2 + \sum_{k=1}^{|\Omega|} E[CF_P^2 | PI = k] \cdot Prob(PI = k) \\
&= -E[CF_P]^2 + \sum_{k=1}^{|\Omega|} p_k \cdot E[CF_{pp_k}^2]
\end{aligned}$$

Way 2

$$\begin{aligned}
Var[CF_P] &= E[(CF_P - E[CF_P])^2] \\
&= \sum_{k=1}^{|\Omega|} E[(CF_P - E[CF_P])^2 | PI = k] \cdot Prob(PI = k) \\
&= \sum_{k=1}^{|\Omega|} E[(CF_{pp_k} - E[CF_P])^2] \cdot p_k \\
&= \sum_{k=1}^{|\Omega|} p_k \cdot E[CF_{pp_k}^2 - 2 \cdot CF_{pp_k} \cdot E[CF_P] + E[CF_P]^2] \\
&= \sum_{k=1}^{|\Omega|} p_k \cdot E[CF_{pp_k}^2] - 2 \cdot E[CF_P] \underbrace{\sum_{k=1}^{|\Omega|} p_k \cdot E[CF_{pp_k}]}_{=E[CF_P]} + E[CF_P]^2 \underbrace{\sum_{k=1}^{|\Omega|} p_k}_{=1} \\
&= -2 \cdot E[CF_P]^2 + E[CF_P]^2 + \sum_{k=1}^{|\Omega|} p_k \cdot E[CF_{pp_k}^2] = -E[CF_P]^2 + \sum_{k=1}^{|\Omega|} p_k \cdot E[CF_{pp_k}^2]
\end{aligned}$$

In the following second step, it is shown how $Var[CF_P]$ can be calculated only by using the expected values and variances of the cash flows of the actions of a process.

Let the probability that both actions $a_d \in AS$ and $a_j \in AS$, with $AS := \bigcup_{k=1}^{|\Omega|} AS_k$, are executed when executing a process be

$$Pr(a_d, a_j) := \mathbf{P}_P \{PP_{a_d, a_j}\} = \sum_{k=1}^{|\Omega|} p_k \cdot \mathbb{I}_{AS_k}(a_d) \cdot \mathbb{I}_{AS_k}(a_j)$$

with the set PP_{a_d, a_j} of process paths, which contains the action a_d as well as the action a_j :

$$PP_{a_d, a_j} = \{pp_k \in \Omega | a_d \in AS_k, a_j \in AS_k\}.$$

Then it is:

$$\begin{aligned} Var[CF_P] &\stackrel{\text{first step}}{\cong} -E[CF_P]^2 + \sum_{k=1}^{|\Omega|} p_k \cdot E[CF_{pp_k}^2] \\ &= -E[CF_P]^2 + \sum_{k=1}^{|\Omega|} p_k \cdot E \left[\left(\sum_{a_d \in AS_k} CF_{a_d} + \sum_{s=1}^S CF_{pa_s} \right)^2 \right] \\ &= -E[CF_P]^2 + \sum_{k=1}^{|\Omega|} p_k \cdot E \left[\left(\sum_{a_d \in AS} CF_{a_d} \cdot \mathbb{I}_{AS_k}(a_d) + \sum_{s=1}^S CF_{pa_s} \right)^2 \right] \\ &= -E[CF_P]^2 \\ &\quad + \sum_{k=1}^{|\Omega|} p_k \\ &\quad \cdot E \left[\left(\sum_{a_d \in AS} CF_{a_d} \cdot \mathbb{I}_{AS_k}(a_d) \right)^2 + 2 \left(\sum_{a_d \in AS} CF_{a_d} \cdot \mathbb{I}_{AS_k}(a_d) \right) \left(\sum_{s=1}^S CF_{pa_s} \right) + \left(\sum_{s=1}^S CF_{pa_s} \right)^2 \right] \\ &= -E[CF_P]^2 \\ &\quad + \sum_{k=1}^{|\Omega|} p_k \\ &\quad \cdot E \left[\sum_{a_d \in AS} CF_{a_d}^2 \cdot \mathbb{I}_{AS_k}(a_d) + \sum_{a_d, a_j \in AS, a_d \neq a_j} CF_{a_d} \cdot CF_{a_j} \cdot \mathbb{I}_{AS_k}(a_d) \cdot \mathbb{I}_{AS_k}(a_j) \right. \\ &\quad \left. + 2 \sum_{a_d \in AS} \sum_{s=1}^S CF_{a_d} \cdot \mathbb{I}_{AS_k}(a_d) \cdot CF_{pa_s} + \sum_{s=1}^S CF_{pa_s}^2 + 2 \sum_{s=1}^{S-1} \sum_{j=s+1}^S CF_{pa_s} \cdot CF_{pa_j} \right] \end{aligned}$$

$$\begin{aligned}
&= -E[CF_P]^2 \\
&+ \sum_{k=1}^{|\Omega|} p_k \left(\sum_{a_d \in AS} E[CF_{a_d}^2] \cdot \mathbb{I}_{AS_k}(a_d) + \sum_{s=1}^S E[CF_{pa_s}^2] \right. \\
&+ \sum_{a_d, a_j \in AS, a_d \neq a_j} E[CF_{a_d} \cdot CF_{a_j}] \cdot \mathbb{I}_{AS_k}(a_d) \cdot \mathbb{I}_{AS_k}(a_j) + 2 \sum_{a_d \in AS} \sum_{s=1}^S E[CF_{a_d} \cdot CF_{pa_s}] \cdot \mathbb{I}_{AS_k}(a_d) \\
&\left. + 2 \sum_{s=1}^{S-1} \sum_{j=s+1}^S E[CF_{pa_s} \cdot CF_{pa_j}] \right) \\
&= -E[CF_P]^2 + \sum_{a_d \in AS} E[CF_{a_d}^2] \cdot \left(\sum_{k=1}^{|\Omega|} p_k \cdot \mathbb{I}_{AS_k}(a_d) \right) + \sum_{s=1}^S E[CF_{pa_s}^2] \cdot \underbrace{\sum_{k=1}^{|\Omega|} p_k}_{=1} \\
&+ \sum_{a_d, a_j \in AS, a_d \neq a_j} E[CF_{a_d} \cdot CF_{a_j}] \cdot \left(\sum_{k=1}^{|\Omega|} p_k \cdot \mathbb{I}_{AS_k}(a_d) \cdot \mathbb{I}_{AS_k}(a_j) \right) \\
&+ 2 \sum_{a_d \in AS} \sum_{s=1}^S E[CF_{a_d} \cdot CF_{pa_s}] \cdot \left(\sum_{k=1}^{|\Omega|} p_k \cdot \mathbb{I}_{AS_k}(a_d) \right) + 2 \sum_{s=1}^{S-1} \sum_{j=s+1}^S E[CF_{pa_s} \cdot CF_{pa_j}] \cdot \underbrace{\sum_{k=1}^{|\Omega|} p_k}_{=1} \\
&= -E[CF_P]^2 + \sum_{a_d \in AS} E[CF_{a_d}^2] \cdot Pr(a_d) + \sum_{s=1}^S E[CF_{pa_s}^2] \\
&+ \sum_{a_d, a_j \in AS, a_d \neq a_j} E[CF_{a_d} \cdot CF_{a_j}] \cdot Pr(a_d, a_j) + 2 \sum_{a_d \in AS} \sum_{s=1}^S E[CF_{a_d} \cdot CF_{pa_s}] \cdot Pr(a_d) \\
&+ 2 \sum_{s=1}^{S-1} \sum_{j=s+1}^S E[CF_{pa_s} \cdot CF_{pa_j}] \\
&= -E[CF_P]^2 + \sum_{a_d \in AS} (Var[CF_{a_d}] + E[CF_{a_d}]^2) \cdot Pr(a_d) + \sum_{s=1}^S (Var[CF_{pa_s}] + E[CF_{pa_s}]^2) \\
&+ \sum_{a_d, a_j \in AS, a_d \neq a_j} (Cov[CF_{a_d}, CF_{a_j}] + E[CF_{a_d}]E[CF_{a_j}]) \cdot Pr(a_d, a_j) \\
&+ 2 \sum_{a_d \in AS} \sum_{s=1}^S (Cov[CF_{a_d}, CF_{pa_s}] + E[CF_{a_d}]E[CF_{pa_s}]) \cdot Pr(a_d) \\
&+ 2 \sum_{s=1}^{S-1} \sum_{j=s+1}^S (Cov[CF_{pa_s}, CF_{pa_j}] + E[CF_{pa_s}]E[CF_{pa_j}])
\end{aligned}$$

$$\begin{aligned}
&= -E[CF_p]^2 + \sum_{a_d \in AS} \left(\text{Var}[CF_{a_d}] + E[CF_{a_d}]^2 \right) \cdot \text{Pr}(a_d) + \sum_{s=1}^S \left(\text{Var}[CF_{pa_s}] + E[CF_{pa_s}]^2 \right) \\
&+ \sum_{a_d, a_j \in AS, a_d \neq a_j} \left(\rho_{a_d, a_j} \cdot \sigma_{a_d} \cdot \sigma_{a_j} + E[CF_{a_d}]E[CF_{a_j}] \right) \cdot \text{Pr}(a_d, a_j) \\
&+ 2 \sum_{a_d \in AS} \sum_{s=1}^S \left(\rho_{a_d, pa_s} \cdot \sigma_{a_d} \cdot \sigma_{pa_s} + E[CF_{a_d}]E[CF_{pa_s}] \right) \cdot \text{Pr}(a_d) \\
&+ 2 \sum_{s=1}^{S-1} \sum_{j=s+1}^S \left(\rho_{pa_s, pa_j} \cdot \sigma_{pa_s} \cdot \sigma_{pa_j} + E[CF_{pa_s}]E[CF_{pa_j}] \right)
\end{aligned}$$

E. Probability of Each Action in Process PR

In order to determine the expected value of CF_{PR} we first need to determine the probability of each action. This is:

$$\begin{aligned}
\text{Pr}(a_1) &= 0.9 + 0.09 + 0.009 + \dots = 0.9 \cdot \sum_{i=0}^{\infty} 0.1^i = 1, \\
\text{Pr}(a_2^{(1)}) &= 0.9 + 0.09 + 0.009 + \dots = 0.9 \cdot \sum_{i=0}^{\infty} 0.1^i = 1, \\
\text{Pr}(a_2^{(2)}) &= 0.09 + 0.009 + 0.0009 + \dots = 0.09 \cdot \sum_{i=0}^{\infty} 0.1^i = 0.1, \\
&\dots, \\
\text{Pr}(a_3^{(1)}) &= 0.9 + 0.09 + 0.009 + \dots = 0.9 \cdot \sum_{i=0}^{\infty} 0.1^i = 1, \\
\text{Pr}(a_3^{(2)}) &= 0.09 + 0.009 + 0.0009 + \dots = 0.09 \cdot \sum_{i=0}^{\infty} 0.1^i = 0.1, \\
&\dots, \\
\text{Pr}(a_4^{(1)}) &= 0.09 + 0.009 + 0.0009 + \dots = 0.09 \cdot \sum_{i=0}^{\infty} 0.1^i = 0.1, \\
&\dots
\end{aligned}$$

Thus, it is for example

$$\sum_{i=1}^{\infty} \text{Pr}(a_2^{(i)}) = \sum_{i=0}^{\infty} 0.1^i = \frac{1}{1-0.1} = \frac{10}{9},$$

which is multiplied with $E[CF_{a_2}]$ since it is $E[CF_{a_2^{(i)}}] = E[CF_{a_2}]$ for all $i \in \mathbb{N}$.

F. Details to determine the variance of CF_{PR}

In order to determine the variance of CF_{PR} with expression (21) it is necessary to calculate $\sum_{a_d, a_j \in AS, a_d \neq a_j} E[CF_{a_d}] E[CF_{a_j}] \cdot Pr(a_d, a_j)$. Hence, we need to determine the probabilities $Pr(a_d, a_j)$. According to expression (15) the process paths and the respective path probabilities need to be calculated. For example there are the process paths

$$pp_1: a_1, a_2^{(1)}, a_3^{(1)}; pp_2: a_1, a_2^{(1)}, a_3^{(1)}, a_4^{(1)}, a_2^{(2)}, a_3^{(2)};$$

$$pp_3: a_1, a_2^{(1)}, a_3^{(1)}, a_4^{(1)}, a_2^{(2)}, a_3^{(2)}, a_4^{(2)}, a_2^{(3)}, a_3^{(3)};$$

$$pp_4: a_1, a_2^{(1)}, a_3^{(1)}, a_4^{(1)}, a_2^{(2)}, a_3^{(2)}, a_4^{(2)}, a_2^{(3)}, a_3^{(3)}, a_4^{(3)}, a_2^{(4)}, a_3^{(4)}, \text{ and}$$

$$pp_5: a_1, a_2^{(1)}, a_3^{(1)}, a_4^{(1)}, a_2^{(2)}, a_3^{(2)}, a_4^{(2)}, a_2^{(3)}, a_3^{(3)}, a_4^{(3)}, a_2^{(4)}, a_3^{(4)}, a_4^{(4)}, a_2^{(5)}, a_3^{(5)},$$

with $p_1 = 0.9$; $p_2 = 0.09$; $p_3 = 0.009$; $p_4 = 0.0009$, and $p_5 = 0.00009$. Considering this five paths Table 5 shows the probabilities $Pr(a_d, a_j)$. For example, the cell in row $a_2^{(1)}$ and column a_1 gives $Pr(a_2^{(1)}, a_1)$. Due to the fact that $Pr(a_d, a_j) = Pr(a_j, a_d)$ it is enough to determine values of the lower triangular table. Since it is $a_d \neq a_j$ in expression (21) and $Pr(a_d, a_d) = Pr(a_d)$ the values on the diagonal do not need to be determined. The process has potentially an infinite number of paths, which means that this table does not contain all relevant probabilities. However, it displays the structure how the values change, which makes it easy to consider all probabilities $Pr(a_d, a_j)$.

	a_1	$a_2^{(1)}$	$a_2^{(2)}$	$a_2^{(3)}$	$a_2^{(4)}$	$a_2^{(5)}$	$a_3^{(1)}$	$a_3^{(2)}$	$a_3^{(3)}$	$a_3^{(4)}$	$a_3^{(5)}$	$a_4^{(1)}$	$a_4^{(2)}$	$a_4^{(3)}$	$a_4^{(4)}$
a_1															
$a_2^{(1)}$	1,0000														
$a_2^{(2)}$	0,1000	0,1000													
$a_2^{(3)}$	0,0100	0,0100	0,0100												
$a_2^{(4)}$	0,0010	0,0010	0,0010	0,0010											
$a_2^{(5)}$	0,0001	0,0001	0,0001	0,0001	0,0001										
$a_3^{(1)}$	1,0000	1,0000	0,1000	0,0100	0,0010	0,0001									
$a_3^{(2)}$	0,1000	0,1000	0,1000	0,0100	0,0010	0,0001	0,1000								

$a_3^{(3)}$	0,0100	0,0100	0,0100	0,0100	0,0010	0,0001	0,0100	0,0100						
$a_3^{(4)}$	0,0010	0,0010	0,0010	0,0010	0,0010	0,0001	0,0010	0,0010	0,0010					
$a_3^{(5)}$	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001				
$a_4^{(1)}$	0,1000	0,1000	0,1000	0,0100	0,0010	0,0001	0,1000	0,1000	0,0100	0,0010	0,0001			
$a_4^{(2)}$	0,0100	0,0100	0,0100	0,0100	0,0010	0,0001	0,0100	0,0100	0,0100	0,0010	0,0001	0,0100		
$a_4^{(3)}$	0,0010	0,0010	0,0010	0,0010	0,0010	0,0001	0,0010	0,0010	0,0010	0,0010	0,0001	0,0010	0,0010	
$a_4^{(4)}$	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001	0,0001

Table 5 Probabilities $\Pr(\mathbf{a}_d, \mathbf{a}_j)$ in Process PR

In Table 5, the values $\Pr(a_d, a_j)$ for the same actions a_d and a_j are encircled. For example, the values in the cells of rows $a_3^{(1)}$ to $a_3^{(5)}$ and column $a_2^{(1)}$ to $a_2^{(5)}$ contain the values for $\Pr(a_d, a_j)$ considering the appearance of the actions a_2 and a_3 in the process paths pp_1 to pp_5 . All of these values have to be considered when calculating $E[CF_{a_d}]E[CF_{a_j}] \cdot \Pr(a_d, a_j)$ in expression (21) for the actions a_2 and a_3 . The different colors show areas with the same structure of the values, to know how to use the formula for a geometric series. With this it is possible to determine $\sum_{a_d, a_j \in AS, a_d \neq a_j} E[CF_{a_d}]E[CF_{a_j}] \cdot \Pr(a_d, a_j)$ in expression (21).

Overall it is

$$\begin{aligned}
& \sum_{a_d, a_j \in AS, a_d \neq a_j} E[CF_{a_d}]E[CF_{a_j}] \cdot \Pr(a_d, a_j) \\
= & \underbrace{\sum_{i=1}^{\infty} E[CF_{a_2^{(i)}}]E[CF_{a_1}] \cdot \Pr(a_2^{(i)}, a_1)}_{\text{grey dashed}} \cdot \underbrace{\sum_{i=2}^{\infty} \sum_{j=1}^{i-1} E[CF_{a_2^{(i)}}]E[CF_{a_2^{(j)}}] \cdot \Pr(a_2^{(i)}, a_2^{(j)})}_{\text{dark grey}} \\
& \text{due to } \Pr(a_d, a_j) = \Pr(a_j, a_d)
\end{aligned}$$

$$\begin{aligned}
& + \underbrace{\sum_{i=1}^{\infty} E[CF_{a_3^{(i)}}] E[CF_{a_1}] \cdot Pr(a_3^{(i)}, a_1)}_{\text{grey dashed}} + \underbrace{\sum_{i=1}^{\infty} E[CF_{a_3^{(i)}}] E[CF_{a_2^{(1)}}] \cdot Pr(a_3^{(i)}, a_2^{(1)})}_{\text{grey dashed}} \\
& + \underbrace{\sum_{i=2}^{\infty} \sum_{j=2}^i E[CF_{a_3^{(i)}}] E[CF_{a_2^{(j)}}] \cdot Pr(a_3^{(i)}, a_2^{(j)})}_{\text{dark grey}} \\
& + \underbrace{\sum_{i=1}^{\infty} \sum_{j=i+1}^{\infty} E[CF_{a_3^{(i)}}] E[CF_{a_2^{(j)}}] \cdot Pr(a_3^{(i)}, a_2^{(j)})}_{\text{black}} \\
& + \underbrace{\sum_{i=2}^{\infty} \sum_{j=1}^{i-1} E[CF_{a_3^{(i)}}] E[CF_{a_3^{(j)}}] \cdot Pr(a_3^{(i)}, a_3^{(j)})}_{\text{dark grey}} \\
& + \underbrace{\sum_{i=1}^{\infty} E[CF_{a_4^{(i)}}] E[CF_{a_1}] \cdot Pr(a_4^{(i)}, a_1)}_{\text{white}} + \underbrace{\sum_{i=1}^{\infty} E[CF_{a_4^{(i)}}] E[CF_{a_2^{(1)}}] \cdot Pr(a_4^{(i)}, a_2^{(1)})}_{\text{white}} \\
& + \underbrace{\sum_{i=1}^{\infty} E[CF_{a_4^{(i)}}] E[CF_{a_2^{(2)}}] \cdot Pr(a_4^{(i)}, a_2^{(2)})}_{\text{white}} + \underbrace{\sum_{i=2}^{\infty} \sum_{j=3}^{i+1} E[CF_{a_4^{(i)}}] E[CF_{a_2^{(j)}}] \cdot Pr(a_4^{(i)}, a_2^{(j)})}_{\text{midium dark grey}} \\
& + \underbrace{\sum_{i=1}^{\infty} \sum_{j=i+2}^{\infty} E[CF_{a_4^{(i)}}] E[CF_{a_2^{(j)}}] \cdot Pr(a_4^{(i)}, a_2^{(j)})}_{\text{light grey}} + \underbrace{\sum_{i=1}^{\infty} E[CF_{a_4^{(i)}}] E[CF_{a_3^{(1)}}] \cdot Pr(a_4^{(i)}, a_3^{(1)})}_{\text{white}} \\
& + \underbrace{\sum_{i=1}^{\infty} E[CF_{a_4^{(i)}}] E[CF_{a_3^{(2)}}] \cdot Pr(a_4^{(i)}, a_3^{(2)})}_{\text{white}} + \underbrace{\sum_{i=2}^{\infty} \sum_{j=3}^{i+1} E[CF_{a_4^{(i)}}] E[CF_{a_3^{(j)}}] \cdot Pr(a_4^{(i)}, a_3^{(j)})}_{\text{midium dark grey}} \\
& + \underbrace{\sum_{i=1}^{\infty} \sum_{j=i+2}^{\infty} E[CF_{a_4^{(i)}}] E[CF_{a_3^{(j)}}] \cdot Pr(a_4^{(i)}, a_3^{(j)})}_{\text{light grey}} + \underbrace{\sum_{i=2}^{\infty} \sum_{j=1}^{i-1} E[CF_{a_4^{(i)}}] E[CF_{a_4^{(j)}}] \cdot Pr(a_4^{(i)}, a_4^{(j)})}_{\text{midium dark grey}} \Big] \\
& = 2 \cdot \left[\underbrace{E[CF_{a_2}] E[CF_{a_1}] \sum_{i=1}^{\infty} Pr(a_2^{(i)}, a_1)}_{\text{grey dashed}} + \underbrace{E[CF_{a_2}] E[CF_{a_2}] \sum_{i=2}^{\infty} \sum_{j=1}^{i-1} Pr(a_2^{(i)}, a_2^{(j)})}_{\text{dark grey}} \right]
\end{aligned}$$

$$\begin{aligned}
& + \underbrace{E[CF_{a_3}]E[CF_{a_1}] \sum_{i=1}^{\infty} \Pr(a_3^{(i)}, a_1)}_{\text{grey dashed}} + \underbrace{E[CF_{a_3}]E[CF_{a_2}] \sum_{i=1}^{\infty} \Pr(a_3^{(i)}, a_2^{(1)})}_{\text{grey dashed}} \\
& + \underbrace{E[CF_{a_3}]E[CF_{a_2}] \sum_{i=2}^{\infty} \sum_{j=2}^i \Pr(a_3^{(i)}, a_2^{(j)})}_{\text{dark grey}} + \underbrace{E[CF_{a_3}]E[CF_{a_2}] \sum_{i=1}^{\infty} \sum_{j=i+1}^{\infty} \Pr(a_3^{(i)}, a_2^{(j)})}_{\text{black}} \\
& + \underbrace{E[CF_{a_3}]E[CF_{a_3}] \sum_{i=2}^{\infty} \sum_{j=1}^{i-1} \Pr(a_3^{(i)}, a_3^{(j)})}_{\text{dark grey}} \\
& + \underbrace{E[CF_{a_4}]E[CF_{a_1}] \sum_{i=1}^{\infty} \Pr(a_4^{(i)}, a_1)}_{\text{white}} + \underbrace{E[CF_{a_4}]E[CF_{a_2}] \sum_{i=1}^{\infty} \Pr(a_4^{(i)}, a_2^{(1)})}_{\text{white}} \\
& + \underbrace{E[CF_{a_4}]E[CF_{a_2}] \sum_{i=1}^{\infty} \Pr(a_4^{(i)}, a_2^{(2)})}_{\text{white}} + \underbrace{E[CF_{a_4}]E[CF_{a_2}] \sum_{i=2}^{\infty} \sum_{j=3}^{i+1} \Pr(a_4^{(i)}, a_2^{(j)})}_{\text{midium dark grey}} \\
& + \underbrace{E[CF_{a_4}]E[CF_{a_2}] \sum_{i=1}^{\infty} \sum_{j=i+2}^{\infty} \Pr(a_4^{(i)}, a_2^{(j)})}_{\text{light grey}} + \underbrace{E[CF_{a_4}]E[CF_{a_3}] \sum_{i=1}^{\infty} \Pr(a_4^{(i)}, a_3^{(1)})}_{\text{white}} \\
& + \underbrace{E[CF_{a_4}]E[CF_{a_3}] \sum_{i=1}^{\infty} \Pr(a_4^{(i)}, a_3^{(2)})}_{\text{white}} + \underbrace{E[CF_{a_4}]E[CF_{a_3}] \sum_{i=2}^{\infty} \sum_{j=3}^{i+1} \Pr(a_4^{(i)}, a_3^{(j)})}_{\text{midium dark grey}} \\
& + \underbrace{E[CF_{a_4}]E[CF_{a_3}] \sum_{i=1}^{\infty} \sum_{j=i+2}^{\infty} \Pr(a_4^{(i)}, a_3^{(j)})}_{\text{light grey}} + \underbrace{E[CF_{a_4}]E[CF_{a_4}] \sum_{i=2}^{\infty} \sum_{j=1}^{i-1} \Pr(a_4^{(i)}, a_4^{(j)})}_{\text{midium dark grey}} \Big] \\
& = 2 \cdot \left[\underbrace{E[CF_{a_2}]E[CF_{a_1}] \sum_{i=1}^{\infty} \Pr(a_2^{(i)}, a_1)}_{\text{grey dashed}} + \underbrace{E[CF_{a_2}]E[CF_{a_2}] \sum_{i=2}^{\infty} \sum_{j=1}^{i-1} \Pr(a_2^{(i)}, a_2^{(j)})}_{\text{dark grey}} \right]
\end{aligned}$$

$$\begin{aligned}
& + \underbrace{E[CF_{a_3}]E[CF_{a_1}] \sum_{i=1}^{\infty} Pr(a_3^{(i)}, a_1)}_{\text{grey dashed}} \\
& + E[CF_{a_3}]E[CF_{a_2}] \left(\underbrace{\sum_{i=1}^{\infty} Pr(a_3^{(i)}, a_2^{(1)})}_{\text{grey dashed}} + \underbrace{\sum_{i=2}^{\infty} \sum_{j=2}^i Pr(a_3^{(i)}, a_2^{(j)})}_{\text{dark grey}} + \underbrace{\sum_{i=1}^{\infty} \sum_{j=i+1}^{\infty} Pr(a_3^{(i)}, a_2^{(j)})}_{\text{black}} \right) \\
& + \underbrace{E[CF_{a_3}]E[CF_{a_3}] \sum_{i=2}^{\infty} \sum_{j=1}^{i-1} Pr(a_3^{(i)}, a_3^{(j)})}_{\text{dark grey}} \\
& + \underbrace{E[CF_{a_4}]E[CF_{a_1}] \sum_{i=1}^{\infty} Pr(a_4^{(i)}, a_1)}_{\text{white}} \\
& + E[CF_{a_4}]E[CF_{a_2}] \left(\underbrace{\sum_{i=1}^{\infty} Pr(a_4^{(i)}, a_2^{(1)})}_{\text{white}} + \underbrace{\sum_{i=1}^{\infty} Pr(a_4^{(i)}, a_2^{(2)})}_{\text{white}} + \underbrace{\sum_{i=2}^{\infty} \sum_{j=3}^{i+1} Pr(a_4^{(i)}, a_2^{(j)})}_{\text{midium dark grey}} \right. \\
& \left. + \underbrace{\sum_{i=1}^{\infty} \sum_{j=i+2}^{\infty} Pr(a_4^{(i)}, a_2^{(j)})}_{\text{light grey}} \right) \\
& + E[CF_{a_4}]E[CF_{a_3}] \left(\underbrace{\sum_{i=1}^{\infty} Pr(a_4^{(i)}, a_3^{(1)})}_{\text{white}} + \underbrace{\sum_{i=1}^{\infty} Pr(a_4^{(i)}, a_3^{(2)})}_{\text{white}} + \underbrace{\sum_{i=2}^{\infty} \sum_{j=3}^{i+1} Pr(a_4^{(i)}, a_3^{(j)})}_{\text{midium dark grey}} \right. \\
& \left. + \underbrace{\sum_{i=1}^{\infty} \sum_{j=i+2}^{\infty} Pr(a_4^{(i)}, a_3^{(j)})}_{\text{light grey}} \right) + \underbrace{E[CF_{a_4}]E[CF_{a_4}] \sum_{i=2}^{\infty} \sum_{j=1}^{i-1} Pr(a_4^{(i)}, a_4^{(j)})}_{\text{midium dark grey}} \\
& = 2 \cdot \left[\underbrace{E[CF_{a_2}]E[CF_{a_1}] \cdot \frac{10}{9}}_{\text{grey dashed}} + \underbrace{E[CF_{a_2}]E[CF_{a_2}] \cdot \frac{1}{9} \sum_{i=0}^{\infty} 0.1^i}_{\text{dark grey}} \right]
\end{aligned}$$

$$\begin{aligned}
& + \underbrace{E[CF_{a_3}]E[CF_{a_1}] \cdot \frac{10}{9}}_{\text{grey dashed}} + E[CF_{a_3}]E[CF_{a_2}] \left(\underbrace{\frac{10}{9}}_{\text{grey dashed}} + \underbrace{\frac{1}{9} \sum_{i=0}^{\infty} 0.1^i}_{\text{dark grey}} + \underbrace{\frac{1}{9} \sum_{i=0}^{\infty} 0.1^i}_{\text{black}} \right) \\
& + \underbrace{E[CF_{a_3}]E[CF_{a_3}] \cdot \frac{1}{9} \sum_{i=0}^{\infty} 0.1^i}_{\text{dark grey}} \\
& + \underbrace{E[CF_{a_4}]E[CF_{a_1}] \cdot \frac{1}{9}}_{\text{white}} + E[CF_{a_4}]E[CF_{a_2}] \left(\underbrace{\frac{1}{9}}_{\text{white}} + \underbrace{\frac{1}{9}}_{\text{white}} + \underbrace{\frac{1}{90} \sum_{i=0}^{\infty} 0.1^i}_{\text{midium dark grey}} + \underbrace{\frac{1}{90} \sum_{i=0}^{\infty} 0.1^i}_{\text{light grey}} \right) \\
& + E[CF_{a_4}]E[CF_{a_3}] \left(\underbrace{\frac{1}{9}}_{\text{white}} + \underbrace{\frac{1}{9}}_{\text{white}} + \underbrace{\frac{1}{90} \sum_{i=0}^{\infty} 0.1^i}_{\text{midium dark grey}} \right. \\
& \left. + \underbrace{\frac{1}{90} \sum_{i=0}^{\infty} 0.1^i}_{\text{light grey}} \right) + \underbrace{E[CF_{a_4}]E[CF_{a_4}] \cdot \frac{1}{90} \sum_{i=0}^{\infty} 0.1^i}_{\text{midium dark grey}} \\
& = 2 \cdot \left[\underbrace{500 \cdot 1,000 \cdot \frac{10}{9}}_{\text{grey dashed}} + \underbrace{500 \cdot 500 \cdot \frac{10}{81}}_{\text{dark grey}} \right. \\
& + \underbrace{500 \cdot 1,000 \cdot \frac{10}{9}}_{\text{grey dashed}} + 500 \cdot 500 \left(\underbrace{\frac{10}{9}}_{\text{grey dashed}} + \underbrace{\frac{10}{81}}_{\text{dark grey}} + \underbrace{\frac{10}{81}}_{\text{black}} \right) + \underbrace{500 \cdot 500 \cdot \frac{10}{81}}_{\text{dark grey}} \\
& + \underbrace{5,000 \cdot 1,000 \cdot \frac{1}{9}}_{\text{white}} + 5,000 \cdot 500 \left(\underbrace{\frac{1}{9}}_{\text{white}} + \underbrace{\frac{1}{9}}_{\text{white}} + \underbrace{\frac{1}{81}}_{\text{midium dark grey}} + \underbrace{\frac{1}{81}}_{\text{light grey}} \right) + 5,000 \\
& \cdot 500 \left(\underbrace{\frac{1}{9}}_{\text{white}} + \underbrace{\frac{1}{9}}_{\text{white}} + \underbrace{\frac{1}{81}}_{\text{midium dark grey}} + \underbrace{\frac{1}{81}}_{\text{light grey}} \right) + \left. \underbrace{5,000 \cdot 5,000 \cdot \frac{1}{81}}_{\text{midium dark grey}} \right] \\
& = 2 \cdot \left[500 \cdot 1,000 \cdot \frac{10}{9} + 500 \cdot 500 \cdot \frac{10}{81} \right. \\
& + 500 \cdot 1,000 \cdot \frac{10}{9} + 500 \cdot 500 \cdot \frac{110}{81} + 500 \cdot 500 \cdot \frac{10}{81} \\
& \left. + 5,000 \cdot 1,000 \cdot \frac{1}{9} + 5,000 \cdot 500 \cdot \frac{20}{81} + 5,000 \cdot 500 \cdot \frac{20}{81} + 5,000 \cdot 5,000 \cdot \frac{1}{81} \right]
\end{aligned}$$

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